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MECH ENG

$$R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{d^2y/dx^2}$$

$x = \cos t + t \sin t$

$y = \sin t - t\cos t$

$dx/dt = -\sin t + t\cos t + \sin t = t\cos t$

$dy/dt = \cos t - (-t\sin t + \cos t)$

$= + t\sin t$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$\frac{d^2y}{dx^2} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$$

$$\frac{dy}{dx} = \frac{\sin t}{\cos t} \quad \left(\frac{u}{v}\right)$$

$dx/dt = \cos t$

$dv/dt = -\sin t$

$$= \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2}$$

$$= \frac{\cos^2 t + \sin^2 t}{(\cos t)^2}$$

Recall that

$\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2}$$

$$= \frac{\left[1 + [\sin t/\cos t]^2\right]^{3/2}}{1/t\cos^3 t}$$

$$= \left(1 + \left[\frac{\sin^2 t}{\cos^2 t}\right]\right)^{3/2} \times t\cos^3 t$$

$$= \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right] \times t\cos^3 t$$

$$= \frac{1}{(\cos^3 t)^{3/2}} \times t\cos^3 t = \frac{1}{\left(\sqrt{\cos^2 t}\right)^3} \times t\cos^3 t$$

$$R = \frac{t\cancel{\cos^3 t}}{\cancel{\cos^3 t}} = t$$

the radius of Curvature is t

b Expressions for the Co-ordinates (h, k) of the Centre of curvature

Recall: $k = x_1 - R\sin\theta$ — ①

$h = y_1 + R\cos^2$ — ②

$R = t, \quad t = \theta$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

Sub x_1 into equ ① and y_1 into equ(2)

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

$(h, k) = (\cos t, \sin t)$