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MECHANICAL ENG
ENG 281 (ENGINEERING MATHS)

1) $\dfrac{(x^2 - \pi/4)\sin(\cos x)}{x \to \pi/2}$

$y = x^2 - \pi/4 \sin(\cos x)$

$u = x^2 - \pi/4$ $\quad$ $V = \sin(\cos x)$

$dy/dx = 2x$ $\quad$ Let $u = \cos x$

$v = \sin$

$dy/dx = dv/du \times dv/dx$

$= \cos u \times -\sin x$

$u = \cos x$

$= -\sin x(\cos x)\cos$

$y = uv$

$dy/dx = u\,dv/dx + v\,du/dx$

$= (x^2 - \pi/4) - \sin x \cos(\cos x) + \sin(\cos(x))\,2x$

$dy/dx - (x^2 - \pi/4)\sin x \cos(\cos x) + 2x\sin(\cos x)$

denominator

$y = x = \pi/2 \quad dy/dx = 1$

$-(x^2 - x/4)\sin\cos(\cos x) + 2x\sin(\cos(x))$

$(\pi/2)^2 - \pi/4 \sin\cos(\cos \pi/2) + 2\pi/2 \sin[\cos \pi/2]$

$\pi = 180$

$(\pi/2)^2 - \pi/4 \sin\cos(\cos 90) + 2(\pi/2)\sin(\cos 90)$

$-(\pi/2)^2 - \pi/4 \sin 90 \cos 0 + 2\pi/2 \sin 0$

$\cancel{(\pi/2)^2 - \pi/2 \sin}$

$-(\pi/2)^2 - \pi/4 \sin 90 \cdot 1 + 2(\pi/2) \times 0$

$-(\pi/2) - \pi/4 (1 \cdot 1) + 0$

$\dfrac{dy}{dx} = -(\pi/2)^2 - \pi/4$

2. $\lim_{x \to \pi/2} \ln\left(\dfrac{(\mathrm{Exp}(3x^2 + 2x + 1)}{x + 1}\right)$

Factoring the Numerator

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$\frac{(3x-1)(x+1)}{(x+1}$

$3\left(\frac{\pi}{2}\right)-1$

$\frac{3\pi}{2}-1$

1c $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right)$

$\cos\left(2\tan^{-1}\left(\frac{\cancel{2}+\sqrt{3}-\cancel{2}}{2+\sqrt{3}-\sqrt{3}}\right)\right)$

$\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\cos(0.8660)$

$\cos 60 = \frac{1}{2}$ //

1d $\lim_{x \to \pi_4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right)$ — undefined

$\lim_{x - x_4} \left(\frac{2x-8}{2x-5}\right)$

$= \frac{8-8}{8-5} = \frac{0}{3} = 0$

3a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6}$

$u_n = \frac{2}{(n+1)(n+2}$ $\quad U_{n+1} = \frac{2}{((n+1)+1)((n+7)+2)}$

$u_{n+1} = \frac{2}{(n+2)(n+3}$

using 1) Albert

$\frac{u_{n+1}}{u_n} = \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$

$\frac{u_{n+1}}{u_n} = \frac{n+1}{n+3}$

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$\frac{U_n+1}{U_n}$

$$\lim_{n\to\infty}\left(\frac{n+1}{n+3}\right) = \left(\frac{n/n + 1/n}{n/n + 3/n}\right) \quad 1/n \to 0 \quad 3/n - 0$$

$$= \frac{1+0}{1-0} = \frac{1}{1} = 1$$

Therefore its inconclusive

b) $\frac{3}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

$U_n = \frac{2}{n^2}$

$U_n+1 = \frac{2}{(n+1)^2}$

$$\frac{U_n+1}{U_n} = \frac{\cancel{2}}{(n+1)^2} \times \frac{n^2}{\cancel{2}}$$

$$\lim_{x\to\infty}\left(\frac{U_n+1}{U_n}\right) = \frac{n^2}{(n+1)^2} \times \frac{n^2}{\cancel{2}}$$

$$\frac{U_n+1}{U_n} = \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} = \frac{1}{1 + \frac{2}{n} + \frac{1}{n^2}} \quad \frac{2}{n} \to 0 \quad \frac{1}{n} \to 0$$

$$\frac{U_n+1}{U_n} = \frac{1}{1+0+0} = \frac{1}{1}$$

Using Comparison test

$\frac{2}{n^2} \quad \frac{1}{n^2}$

$\frac{2}{1^2} \;\angle\; \frac{1}{1^2}$

2c $\frac{U_n + 2n^2}{1 + n^2}$

$$U_n = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + \frac{n^2}{n^2}} \qquad \frac{1}{n^2} \to 0$$

$$\frac{1}{n^2} \to 0$$

$$= \frac{0+2}{0+1} = \frac{2}{1}$$

Therefore the Series is Divergent

3 $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{2(n+1)+1)^2}$$

$$U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{2(n+2)^3} \times \frac{2(n+1)^3}{x^n}$$

$$\frac{x^n \cdot x^1}{2(n+2)^3} \times \frac{2(n+1)^3}{x^n}$$

$$= \frac{x(n+1)^3}{(n+2)^3}$$

Expanding the bracket

$$x\left(\frac{n^3 + 3n^2 + 3n + 1}{n^3 + 6n^2 + 12n + 8}\right)$$

$$\to x$$

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$$= x\left(\frac{\frac{n^3}{n^3}+\frac{3n^2}{n^3}+\frac{3n}{n^3}+\frac{1}{n^3}}{\frac{n^3}{n^3}+\frac{6n^3}{n^3}+\frac{12n}{n^2}+\frac{6}{n^3}}\right)$$

$$= x\left(\frac{1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}}{1+\frac{6}{n}+\frac{12}{n^2}+\frac{6}{n^3}}\right)$$

$$= x\left(\frac{1+0+0+0}{1+0+0+0}\right)$$

$$x\left(\frac{1}{1}\right) = x$$

$-1 < x < 1$

for $x \leq 1$

$-1 < x < 1$

$$U_n = \frac{1^n}{(2n+1)^3} < \frac{1}{n^3}$$

$\therefore -1 < n \leq 1$

for $x = -1$

$$\frac{-1}{(2n+3)^2} < \frac{1}{n^3}$$

$$\frac{1}{(2n+3)^3} > -\frac{1}{n^3}$$

$\therefore -1 \leq n \leq 1$

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4 $\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

$\left(\frac{\sin(0) - \cos(0)}{0^3} \right) \to$ undefined

$\frac{d}{dx}\left(\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) \right) = \frac{\cos x + \sin x}{3x^2} - 0$

$\frac{d}{dx}\left(\lim_{x \to 0} \left(\frac{\cos x + \sin x}{3x^2} \right) \right) = \left(\frac{-\sin x + \cos x}{6x} \right) - 0$

$\frac{d}{dx}\left(\lim_{x \to 0} \left(\frac{-\sin x + \cos x}{6x} \right) \right) = \left(\frac{-\cos x - \sin x}{6} \right)$

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$\therefore \frac{-\cos(0) - \sin(0)}{6} = \frac{-1}{6}$