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16/ENGG02/050

Computer Engineering

ENGG 281

Assignment 2

The parametric equation of the curve are given

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

To find the radius of curvature (κ) in terms of t

$$\frac{dx}{dt} = -\sin t + (t \cos t + \sin t)$$

$$= -\sin t + t \cos t + \sin t$$

$$\frac{dx}{dt} = t \cos t$$

$$\frac{dy}{dt} = \cos t - (t - \sin t + \cos t)$$

$$= \cos t + t \sin t - \cos t$$

$$= t \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t \sin t}{1} \times \frac{1}{t \cos t}$$

$$\frac{dy}{dx} = \frac{t \sin t}{t \cos t}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left[\frac{t \sin t}{t \cos t} \right] \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{(t \cos t) \cdot (t \cos t + t \sin t) - (t \sin t) \cdot (-t \sin t + \cos t)}{(t \cos t)^2 \times \frac{dt}{dx}}$$

$$= \frac{t^2 \cos^2 t + t \cos t \sin t + t^2 \sin^2 t - t \cos t \sin t}{(t \cos t)^2} \times \frac{1}{t \cos t}$$

$$= \frac{t^2 \cos^2 t + t^2 \sin^2 t}{(t \cos t)^2} \times \frac{1}{t \cos t}$$

$$= \frac{t^2 (\cos^2 t + \sin^2 t)}{t^2 \cos^2 t} \times \frac{1}{t \cos t}$$

Recall $\cos^2 t + \sin^2 t = 1$

$$= \frac{t^2 \cdot 1}{t^2 \cos^2 t} \times \frac{1}{t \cos t}$$

$$\frac{d^2 y}{dx^2} = \frac{1}{t \cos^3 t}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{d^2 y / dx^2}$$

$$= \frac{\left[1 + \left[\frac{t \sin t}{t \cos t} \right]^2 \right]^{3/2}}{\frac{1}{t \cos^3 t}}$$

$$= \left[\frac{1}{1} + \frac{t^2 \sin^2 t}{t^2 \cos^2 t} \right]^{3/2} \times \frac{t \cos^3 t}{1}$$

$$= \left[\frac{t^2 \cos^2 t + t^2 \sin^2 t}{t^2 \cos^2 t} \right]^{3/2} \times \frac{t \cos^3 t}{1}$$

$$= \left[\frac{t^2}{t^2 \cos^2 t} \right]^{3/2} \times t \cos^3 t$$

$$= \left[\frac{1}{\cos^2 t} \right]^{3/2} \times t \cos^3 t$$

$$= \left[\frac{\sqrt{1}}{\cos t} \right]^3 \times \frac{t \cos^3 t}{1}$$

$$R = t$$

Therefore, the radius of curvature in terms of t is $R = t$

For the coordinates of the centre of curvature (h, k)

$$h = x_1 - R \sin \theta$$

$$k = y_1 + R \cos \theta$$

$$\tan \theta = \frac{dy}{dx} = \frac{t \sin t}{t \cos t} = \tan t$$

$$\theta = \tan^{-1}(\tan t)$$

$$\theta = t$$

$$x_1 = \cos t + t \sin t \text{ from eqn (i)}$$

$$y_1 = \sin t - t \cos t \text{ from eqn (ii)}$$

$$h = \cos t + t \sin t - [t][\sin t]$$

$$h = \cos t + t \sin t - t \sin t$$

$$h = \cos t$$

$$k = \sin t - t \cos t + [t][\cos t]$$

$$k = \sin t - t \cos t + t \cos t$$

$$k = \sin t$$

Therefore the expressions in terms of the centre of curvature (h, k) are

$$h = \cos t \text{ and } k = \sin t$$