$$\lim_{x=\pi/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{x-\pi/2}\right] = \lim_{x=\pi/2}\left[\frac{(x^2-\pi/4)(-\sin x\cos(\cos x))+2x\sin(\cos x)}{x}\right]$$

$$= \frac{\left(\left(\frac{\pi}{2}\right)^2-\frac{\pi}{4}\right)\left(-\sin\frac{\pi}{2}\cos\left(\cos\frac{\pi}{2}\right)+\left(2\left(\frac{\pi}{2}\right)\sin\left(\cos\frac{\pi}{2}\right)\right)\right]}{\pi/2}$$

$$= \frac{\left[\frac{\pi^2}{4}-\frac{\pi}{4}\right]\left[-x\cos(0)\right]+\left[\pi\sin(0)\right]}{\pi/2}$$

$$\frac{\left(\frac{\pi^2-\pi}{4}\right)(-1)+0}{\pi/2}$$

$$= \frac{-\pi^2+\pi}{4} \div \frac{\pi}{2} = \frac{-\pi^2+\pi}{4} \times \frac{2}{\pi}$$

$$= \frac{\pi(-\pi+1)}{2} \times \frac{1}{\pi}$$

$$\lim_{x=\pi/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{x-\pi/2}\right] = \frac{1-\pi}{2}$$

(1b) $$\lim_{x=\pi/2}\ln\left[e^{\frac{(3x^2+2x-1)}{x+1}}\right] = \lim_{x=\pi/2}\left[\frac{3x^2+2x-1}{x+1}\right]$$

$$= \lim_{x=\pi/2}\frac{(3x-1)(x+1)}{x+1}$$

$$= \lim_{x=\pi/2}(3x-1)$$

$$= 3\left(\frac{\pi}{2}\right)-1$$

$$\lim_{x=\pi/2}\ln\left[e^{\frac{(3x^2+2x-1)}{x+1}}\right] = \frac{3\pi}{2}-1$$

(1c) $$\lim_{x=2+\sqrt{3}}\left[\cos\left[\sin^{-1}\left(\frac{x-2}{x-\sqrt{3}}\right)\right]\right] = \cos\left(\sin^{-1}\left(\frac{2+\sqrt{3}-2}{2+\sqrt{3}-\sqrt{3}}\right)\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \cos 60$$

$$\lim_{x=2+\sqrt{3}}\left[\cos\left[\sin^{-1}\left(\frac{x-2}{x-\sqrt{3}}\right)\right]\right] = \frac{1}{2}$$

3rd of Oct 2017
MAUFUMAU YUSUF ADEMOLA
16/ENG06/039
MECHANICAL ENG
ENG 281

1) Evaluate the limits of functions

a) $\lim_{x = \pi/2} \left[\frac{(x^2 - \pi/4)\, \sin(\cos x)}{x - \pi/2} \right]$ = indefinite

$\lim_{x = \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos)x}{x - \pi/2} \right] = \lim_{x \to \pi/2} \frac{\left[d/dx \left[(x^2 - \pi/4) \sin(\cos x)\right]\right]}{\left[d/dx \, (x - \pi/2) \right]}$

$\frac{d}{dx} \left[(x^2 - \pi/4) \sin(\cos x) \right] = u\frac{dv}{dx} + v\frac{du}{dx}$

Let $u = x^2 - \pi/4$

$\frac{du}{dx} = 2x$

Let $v = \sin(\cos x)$

Let $w = \cos x$

$Y = \sin w$

$\frac{dy}{dw} = \cos w$

$dw/dx = \cos x$

$\frac{dx}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dx}$

$= \cos w \, (-\sin x)$

$\frac{dy}{dx} = \cos(\cos x)(-\sin x)$

$\frac{dy}{dx} = -\sin x \cos(\cos x)$

$\frac{d}{dx} (x^2 - \pi/4) \sin(\cos x)] = (x^2 - \pi/4)(-\sin x \cos(\cos x) + \sin(\cos x)($

2b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots$

$U_n = \frac{2}{n^2}$ U_{n+1} $\frac{2}{(n+1)^2}$

$\lim_{n=\infty} = \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2}$

$\lim_{n=\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \div n^2$

$= \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$

$\lim_{n=\infty} = \frac{n^2/n^2}{n^2/n^2 + 2n/2n^2 + 1/n^2} = \frac{1}{1 + 2/n + 1/n^2} = \frac{1}{1} = 1$

from test 1

$\lim_{n=\infty} U_n \lim_{n=\infty} \lim_{n=\infty} = \frac{2}{n^2}$

$\lim_{n=\infty} = \frac{2/n^2}{n^2/n^2} = \frac{0}{1} = 0$

$\therefore$ the series ~~of~~ is convergent

2c $U_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n=\infty} = \frac{1+2n^2}{1+n^2} = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$

$= \frac{0+2}{1} = 2$

The series is divergent

3) find the range of values of x for which the series below absolutely convergent

$$\frac{x}{1\,2\,7} + \frac{x^2}{1\,2\,5} \cdots + \frac{x^n}{(2n+1)^3}$$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\lim \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \left(\frac{2n+1}{x^n}\right)^3$$

$$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{8n^3+12n^2+n+1}{8n^3+24n^2+24n+8}$$

Dividing by n^3

$$= \frac{\left(8+{}^{2}/_{n}+{}^{1}/_{n^2}+{}^{1}/_{n^3}\right)}{\left(8+{}^{12}/_{n}+{}^{24}/_{n^2}+{}^{8}/_{n^3}\right)}$$

as $n = \infty$, $1/n = 0$

$$\frac{8x}{8} \geq x-1, \quad x<1$$

4) Evaluate Using L'Hospital's Rule.

$$\lim_{x=0}\left(\frac{\sin x - \cos x}{x^3}\right)$$

$$\lim_{x=0}\left(\frac{\sin x - \cos x}{x^3}\right) = \lim_{x=0}\left(\frac{\cos x + \sin x}{3x^2}\right)$$

$$= \lim_{x=0}\left(\frac{-\sin x + \cos x}{6x}\right)$$

$$= \lim_{x=0}\left(\frac{-\cos x - \sin x}{6}\right)$$

$$= \frac{-\cos 0 - \sin 0}{6}$$

$$= \frac{-\cos 0 - \sin 0}{6}$$

$$= \frac{-1-0}{6}$$

$$= -1/6$$

(d) $\lim_{x=4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right) = \lim_{x=4} \left(\frac{x-4}{x-1} \cdot \cancel{\frac{x-4}{x-4}}\right)$

$= \lim_{x=4} \left(\frac{x-4}{x-1}\right)$

$= \frac{4-4}{4-1} = 0$

$\lim_{x-4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right) = 0_{//}$

2a) Determine whether each of the ffg series is ~~converg~~

$\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

$U_n = \frac{2}{(n+1)(n+2}$ $\qquad U_{n+1} = \frac{2}{(n+2)(n+3)}$

$\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{3}$

$= \frac{n+1}{n+3}$

$\lim_{n=\infty} \frac{U_{n+1}}{U_n} = \lim_{n=\infty} \frac{n+1}{n+3}$

$= \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}} = \frac{1+\frac{1}{n}}{1+\frac{3}{n}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$

Since $\lim_{n=\infty} \frac{U_{n+1}}{U_{n+1}}$

The series is inconclusive