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ENG 281

Assignment 2.

The parametric equations of a curve are given in eqn (1) and (2)

$$x = \cos t + t\sin t$$

$$y = \sin t - t\cos t$$

In terms of t, determine:

i) An expression for the radius of curvature (R), and

ii) Expressions for the coordinates (h, k) of the Centre of Curvature

1.) $$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$\frac{dy}{dx} = ?$, $\frac{d^2y}{dx^2} = ?$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$y = \sin t - t\cos t$$

$$\frac{dy}{dt} = \cos t - [-t\sin t + \cos t]$$

$$= \cos t + t\sin t - \cos t$$

$$\frac{dy}{dt} = t\sin t$$

$$x = \cos t + t\sin t$$

$$\frac{dx}{dt} = -\sin t + (t\cos t + \sin t)$$

$$= -\sin t + t\cos t + \sin t$$

$$\frac{dx}{dt} = t\cos t$$

$$\frac{dy}{dx} = t\sin t \div t\cos t$$

$$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$= \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2} \times \frac{dt}{dx}$$

$$= \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$$

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$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$= \left[1 + \left(dy/dx\right)^2\right]^{3/2} \div d^2y/dx^2$$

$$= \left[\frac{1}{1} + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$\left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left(\frac{1}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$= \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$= \frac{t\cos^3 t}{\cos^3 t}$$

$$R = t$$

ii.) Centre of Curvature.

$h = x - R\sin\theta$ ------ (1)

$k = y + R\cos\theta$ ----- (2)

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$R = t, \theta = t$

Sub x, R, θ into eqn (1) and y, R, θ in to eqn (2)

$h = (\cos t + t\sin t) - t\sin t$

$h = \cos t$

$k = (\sin t - t\cos t) + t\cos t$

$k = \sin t$

$(h, k) = [\cos t, \sin t]$