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① The parametric

Solution

1a) $R = \dfrac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$

$x \Rightarrow \cos t + t\sin t$

$\frac{dx}{dt} \Rightarrow -\sin t + t\cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t - (-t\sin t + \cos t)$

$= \cos t + t\sin t - \cos t$

$= t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= t\sin t \times \frac{1}{t\cos t} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$

$$\frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{v\frac{dy}{dx} - u\frac{dy}{dx}}{v^2}$$

where $v = \cos t$

$du = \sin t$

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$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

Recall that $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \cdot \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

Also recall that

$$R = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

$$R = \frac{\left\{1 + \left(\frac{\sin t}{\cos t}\right)^2\right\}^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{\left\{1 + \frac{\sin^2 t}{\cos^2 t}\right\}^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$\rho = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times \frac{t\cos^3 t}{1}$$

$$\rho = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2} \times t\cos^3 t.$$

$$\rho = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$= \frac{1}{(\sqrt{\cos^2 t})^3} \times t\cos^3 t$$

$$\rho = \frac{t\cos^3 t}{(\sqrt{\cos^3 t})^3}$$

$$\rho = t$$

(b) $h = x_1 - \rho\sin\theta - 1$

$k = y_1 + \rho\cos\theta - 2$

$\rho = t \quad t = \theta$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

Substituting x_1 in equ 1 and y_1 in equ 2

$h = \cos t + t\sin t - t\sin t$

$h = \cos t.$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

$\therefore \quad (h, k) = (\cos t, \sin t).$