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15/ENG07/008

CHEMICAL ENGINEERING

ENG 381: ENGINEERING MATHEMATICS

ASSIGNMENT TWO

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CHEMICAL ENGINEERING
ENG 351:- ENGINEERING MATHEMATICS III

ASS

1) The model of a system is given as an Equation as an Equation (0.1)

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

i Obtain an Equation for y as a function of θ

SOL

Let $6\sin\theta = 0$

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = -2 \pm i$$

Hence

Complimentary function; $y = e^{-2\theta}(A\cos\theta + B\sin\theta)$.

To obtain our particular integral

Let $y = C\cos\theta + D\sin\theta$

$$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$$

$$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$$

Substitute all of the above in the original Equation.

$$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$$

$$-C\cos\theta + 4D\cos\theta + 5C\cos\theta - D\sin\theta - 4C\sin\theta + 5D\sin\theta = 6\sin\theta$$

$$\cos\theta(4C + 4D) + \sin\theta(4D - 4C) = 6\sin\theta$$

Comparing Coefficient

$$4D - 4C = 6 \quad \cdots (i)$$

$$4D + 4C = 0 \quad \cdots (ii)$$

$$8D = 6$$

$$D = \frac{6}{8} = \frac{3}{4}$$

Substitute $D = \frac{3}{4}$ into Eq. (i)

$$4\left(\frac{3}{4}\right) - 4C = 6$$

$$-4C = 6 - 3$$

$$-4C = 3$$

$$C = -\frac{3}{4}$$

$\therefore$ Our P.I : $y = C\cos\theta + D\sin\theta$

$$y = \frac{3}{4}\cos\theta - \frac{3}{4}\sin\theta$$

$\therefore$ G.S = P.I + C.F

$$y = \frac{3}{4}\cos\theta - \frac{3}{4}\sin\theta + e^{-2\theta}(A\cos\theta + B\sin\theta)$$

=

iii) Rate of Steady State $\frac{dy}{d\theta} = 0$

Now $\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$$0 = -C\sin\theta + D\cos\theta$$

$$-\frac{3}{4}\sin\theta = \frac{3}{4}\cos\theta$$

$$\frac{-\frac{3}{4}\sin\theta}{\frac{3}{4}} = \frac{\frac{3}{4}\cos\theta}{\frac{3}{4}}$$

$$-1 \cdot \frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = -1$$

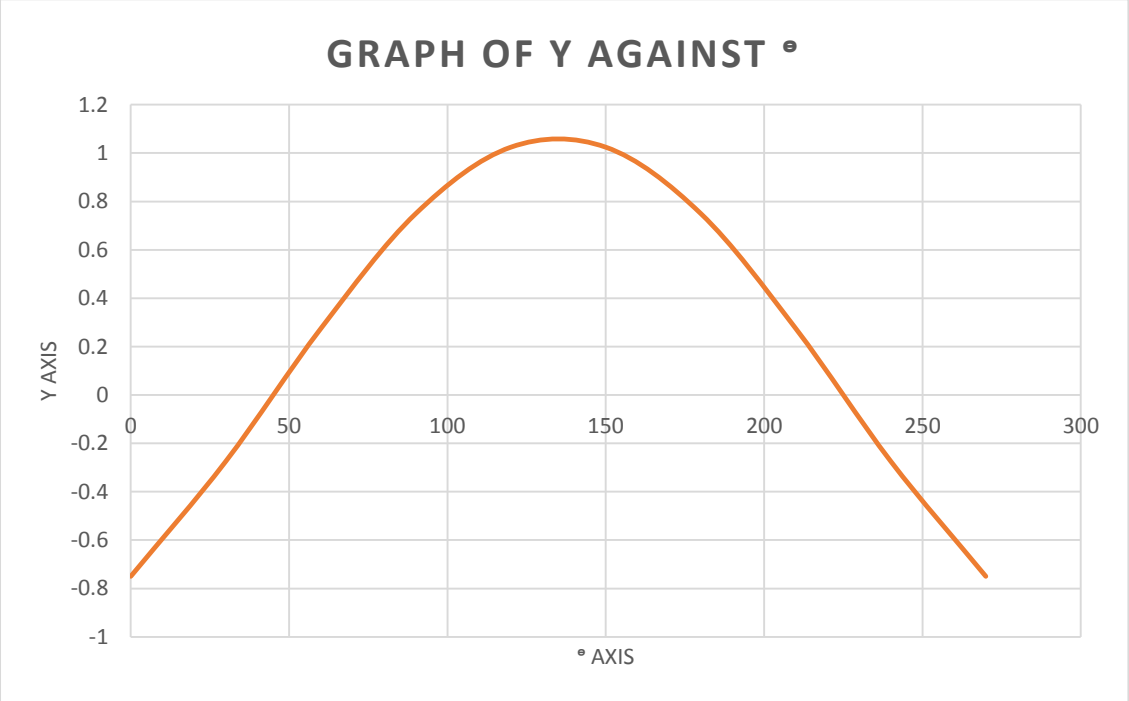
$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

=

Hence the value of $\theta = -45^\circ$

ɵ	y
0	-0.75
30	-0.27452
60	0.274519
90	0.75
120	1.024519
150	1.024519
180	0.75
210	0.274519
240	-0.27452
270	-0.75



2) $EI\frac{d^2y}{dx^2} = \frac{W}{2}(l-x)^2$

$EI\frac{d^2y}{dx^2} = \frac{W}{2}(l^2 - 2lx + x^2)$

Let R.H.S = 0

$EI\frac{dy}{dx^2} = 0$

$m^2 = 0$

$m = \pm 0$

Hence our Complimentary function is given as

$y = e^{0x}(A + Bx)$

$y = A + Bx$

To obtain our particular integral.

$y = Kx^2 + Lx^3 + mx^4$ — l = Length of beam.

$\frac{dy}{Ix} = 2Kx + 3Lx^2 + 4mx^3$ — L = Constant.

$\frac{d^2y}{dx^2} = 2K + 6Lx + 12mx^2$.

Substituting all of the above into the original equation.

$EI(2K + 6Lx + 12mx^2) = \frac{W}{2}(l^2 - 2lx + x^2)$

$2(2KEI + 6LEIx + 12EImx^2) = W(l^2 - 2lx + x^2)$

$4KEI + 12LEIx + 24EImx^2 = W(l^2 - 2lx + x^2)$

Comparing Co-efficient

$W = 24EIm$

$m = \frac{W}{24EI}$ ... (A)

$12LEIx = -2Wl$

$L = \frac{-2Wl}{12EI} = \frac{-Wl}{6EI}$ ... (A*)

$4KEI = 0$ $\quad 4KEI = Wl^2$

$k = \frac{Wl^2}{4EI}$ ... (A**)

Solving on rt the above in the original equation

$$y = \left(\frac{WL^2}{4EI}\right)x^2 - \left(\frac{WL}{6EI}\right)x^3 + \left(\frac{W}{24EI}\right)x^4$$

$$y = \frac{WL^2x^2}{4EI} - \frac{WLx^3}{6EI} + \frac{Wx^4}{24EI}$$

$$y = \frac{6WL^2x^2 - 4WLx^3 + Wx^4}{24EI}$$

$$y = \frac{W}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right].$$

Apply our GS;

$$y = P.I + C.F$$

$$y = A + Bx + \frac{W}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

When $y=0$, $x=0$, $\frac{dy}{dx} = 0$.

$$0 = A + B(0) + \frac{W}{24EI}\left[6L^2(0) - 4L(0)^3 + 0^4\right]$$

$$A = 0.$$

For B,

$$\frac{dy}{dx} = B + \frac{W}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{W}{24EI}\left[12L^2(0) - 12(0)^2 + 4(0)^3\right]$$

$$0 = B$$

$$\therefore B = 0$$

Now to obtain the value expression in y when $x = L$.

$$y = A + Bx + \frac{W}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = v + a(x) + \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = \frac{w}{24EI}\left(6L^4 - 4L^4 + L^4\right)$$

$$y = \frac{w}{24EI} \cdot 3L^4$$

$$y = \frac{wL^4}{8EI} \qquad x = L$$