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The parametric equations of a curve are given in equations 1 and 2

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

In terms of t determine

(i) An expression for the radius of curvature (R) and

(ii) An expression for the coordinates (h, k) of the centre of curvature.

Solution

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$dy/dt = -\sin t + t\cos t + \sin t = t\cos t$

$dy/dt = \cos t - (-t\sin t + \cos t)$

$dy/dt = \cos t + t\sin t - \cos t = t\sin t$

$dy/dx = dy/dt \times dt/dx$

$dy/dx = t\sin t \times 1/t\cos t$

$dy/dx = t\sin t / t\cos t$

$d^2y/dx^2 = \frac{V\,dy/dx - U\,dv/dx}{V^2}$

$U = t\sin t \qquad V = t\cos t$

$dy/dx = -t\cos t + \sin t \qquad dv/dx = -t\sin t + \cos t$

$\therefore \frac{d^2y}{dx^2} = \frac{(t\cos t)\cdot(t\cos t + \sin t) - (t\sin t)\cdot(-t\sin t + \cos t)}{(t\cos t)^2} \times \frac{1}{t\cos t}$

$= \frac{t^2\cos^2 t + t\cos t\sin t - (-t^2\sin^2 t + t\cos\sin t)}{(t\cos t)^2} \times \frac{1}{t\cos t}$

$\frac{d^2y}{dx^2} = \frac{t^2\cos^2 t + t\cos t\cdot\sin t + t^2\sin^2 t - t\cos t\sin t}{(t\cos t)^2} \times \frac{1}{t\cos t}$

$$= \frac{t^2\cos^2 t + t^2\sin^2 t}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2(\cos^2 t + \sin^2 t)}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2}{t^2\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + \left(\frac{t\sin t}{t\cos t}\right)^2\right]^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \left[\frac{1 + \frac{t^2\sin^2 t}{t^2\cos^2 t}}{1}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \left[\frac{t^2(\cos^2 t + \sin^2 t)}{t^2\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

But $\cos^2 t + \sin^2 t = 1$

$$R = \left[\frac{t^2}{t^2\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{1}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{\sqrt{1}}{\sqrt{\cos^2 t}}\right]^3 \times t\cos^3 t$$

$$= \frac{1}{(\cos t)^3} \times t\cos^3 t$$

(ii)

$h = x - R\sin t$

$k = y + R\cos t$

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$h = \cos t + t\sin t - t\sin(t)$

$h = \cos t,$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t,$