6/10/17

(a) 1) Evaluate: $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \cdot (\sin(\cos x))}{x - \frac{\pi}{2}} \right]$

Solution

Applying product's rule; $U\frac{dv}{dx} + V\frac{du}{dx}$
for the numerator

$U = x^2 - \pi/4$, $V = \sin(\cos x)$

$a = \cos x \Rightarrow V = \sin a$

$\frac{da}{dx} = -\sin x .$ $\frac{dv}{da} = \cos a = \cos(\cos x)$

$\frac{dv}{da} \times \frac{da}{dx} = \frac{dv}{dx} = -\sin x \cdot \cos(\cos x)$

$\frac{dv}{dx} = -\sin x \cdot \cos(\cos x)$.

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(-\sin x \cdot \cos x) + \sin(\cos x)(2x)}{x - \frac{\pi}{2}} \right]$

$= \frac{\left[\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \right] \cos(\cos \pi/2) \cdot \sin x + 2(\pi/2)\sin(\cos \pi/2)}{1}$

$= \frac{\pi \sin 0 + \left[\frac{\pi^2}{4} - \frac{\pi}{4} \right] \cos 0 \cdot -\sin \frac{\pi}{2}}{1}$

(2)

$= 0 + \left[\frac{\pi^2}{4} - \frac{\pi}{4}\right] 1 \cdot -1$

$= \underline{-\frac{\pi^2}{4} + \frac{\pi}{4}}$

1b) Evaluate: $\lim_{x \to \pi/2} \ln\left[\exp \frac{(3x^2+2x-1)}{x+1}\right]$

__Solution__

Factorizing $(3x^2+2x-1)$

$3x^2+3x-x-1 = 3x(x+1)-1(x+1)$

$= (3x-1)(x+1)$

$\exp\frac{(3x^2+2x-1)}{x+1} = \exp\frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$

$= \exp(3x-1)$

$\Rightarrow \lim_{x \to \pi/2} \ln\left[\exp(3x-1)\right]$

$= \lim_{x \to \pi/2} \ln\left[\exp\left(3(\pi/2)-1\right)\right]$

$= \lim_{x \to \pi/2} \ln\left[\exp\left(3(\pi/2)-1\right)\right]$

$\cancel{\ln \exp} = \ln(40.9515)$

$= \underline{3.7124}$

1c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1} \frac{(x-2)}{x-\sqrt{3}}\right]$

Solution

$\left[\sin^{-1} \frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$= \left[\sin^{-1} \frac{(\sqrt{3})}{(2)}\right] = 60$

$\cos(60) = 1/2$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$

1d) $\lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4}\right]$: Evaluate

Solution

$= \frac{[(x-4)\cancel{(x-4)}]}{(x-1)\cancel{(x-4)}}$

$\lim_{x \to 4} \left[\frac{(x-4)}{(x-1)}\right] = \frac{4-4}{4-1} = \frac{0}{3} = 0$

2a) $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \cdots$

Comparing this series to a standard series that converges,

$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \cdots$

when $p=2$.

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\cdots$$

$$\frac{2}{1\times3}<\frac{1}{1^2},\quad \frac{1}{2\times3}<\frac{1}{2^2},\quad \frac{2}{4\times5}<\frac{1}{3^2},\quad \frac{2}{5\times6}<\frac{1}{4^2}.$$

Each term of the given series is less than the corresponding term in the series known to converge. Therefore, the given series converges.

2b) $$\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\cdots$$

Comparing this series to a standard series that converges

$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\frac{1}{5^p}+\frac{1}{6^2}+\cdots$$

when $p=2$

$$\frac{1}{1^p}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\cdots$$

$$\frac{2}{12}>\frac{1}{1^2},\quad \frac{2}{2^2}>\frac{1}{2^2},\quad \frac{2}{3^2}>\frac{1}{3^2},\quad \frac{2}{4^2}>\frac{1}{4^2}$$

* Each term of the given series is greater than the corresponding term in the series known to converge. Therefore, the given series diverges.

(5)

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(2c)

$U_n = \frac{1+2n^2}{1+n^2}$

$U_{n+1} = \frac{1+2(n+1)^2}{(n+1)^2+1}$

2b

$\frac{U_{n+1}}{U_n} = \frac{1+2(n+1)^2}{(n+1)^2+1} \times \frac{n^2+1}{1+2n^2}$

$= \frac{1+2n^2}{1+n^2} = \frac{1+2(n+1)^2}{1+(n+1)^2} \quad (n = n+1)$

$= \frac{1+2n^2+4n+2}{1+n^2+2n+1} \times \frac{1+n^2}{1+2n^2}$

$= \frac{(2n^2+4n+3)(1+n^2)}{(n^2+2n+2)(1+2n^2)}$

$= \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^2}$

$= \frac{5n^2+2n^4+4n^3+4n+3}{5n^2+2n^4+4n^3+2n+2}$

$\lim_{n\to\infty} = \left\{ \frac{5/n^2+2+4/n+4/n^3+3/n^4}{5/n^2+2+4/n+2/n^3+2/n^4} \right\}$

$= \frac{2}{2} = 1$ (Inconclusive)

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3) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{[2(n+1)+1]^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n} = \frac{x \cdot x^n}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$\frac{U_{n+1}}{U_n} = \frac{x(2n+1)^3}{(2n+3)^3} = x\frac{(8n^3+12n^2+6n+1)}{(8n^3+36n^2+54n+27)}$$

$\boxed{-1 < x \leq 1 \text{ Ans}}$

$$\frac{U_{n+1}}{U_n} = x\frac{\left(8 + \frac{12}{n} + \frac{6}{n^2} + \frac{1}{n^3}\right)}{8 + \frac{36}{n} + \frac{54}{n^2} + \frac{27}{n^3}}$$

$$n \to \infty = \frac{x(\cancel{8})}{\cancel{8}} = x$$

$$U_\infty = \frac{U_{n+1}}{U_n} = x$$

$\Rightarrow -1 < x \leq 1$ is the range of value of x.

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4. $\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

sub $x = 0$

$= \left(\frac{\sin 0 - \cos 0}{0} \right)$

$= \frac{0}{0}$ = Undefined

$\frac{d}{dx} \left(\frac{\sin x - \cos x}{x^3} \right) = \frac{\cos x + \sin x}{3x^2}$

sub $x = 0$

$= \frac{\cos 0 + \sin 0}{3(0)} = \frac{1+0}{0}$, Undefined

$\frac{d}{dx} \left(\frac{\cos x + \sin x}{3x^2} \right) = \frac{-\sin x + \cos x}{6x}$

sub $x = 0$

$\frac{-\sin(0) + \cos(0)}{6(0)} = \frac{-0+1}{0}$ = Undefined

$\frac{d}{dx} \left(\frac{-\sin x + \cos x}{6x} \right) = \frac{-\cos x - \sin x}{6}$

sub $x = 0$

$\frac{-\cos(0) - \sin(0)}{6} = \frac{-1-0}{6} = \frac{-1}{6}$