15/ENG06/015
Awurum I. Charles
Mechanical Engineering
Assignment 2

(1) $\frac{d^2y}{dx^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

Solution

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$

$m = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm \sqrt{4}.\sqrt{-1}}{2}$

$m = \frac{-4}{2} \pm \frac{2i}{2}$

$m = -2 \pm j$

$y = e^{-2\theta}[C\cos\theta + D\sin\theta]$

Assumed PI

$y = A\cos\theta + B\sin\theta$

$\frac{dy}{d\theta} = -A\sin\theta + B\cos\theta$

$\frac{d^2y}{d\theta^2} = -A\cos\theta - B\sin\theta$

$(-A\cos\theta - B\sin\theta) + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$(-A + 4B + 5A)\cos\theta + (-B - 4A + 5B)\sin\theta = 6\sin\theta$

$(4A+4B)\cos\theta + (4A+4B)\sin\theta = 6\sin\theta$

$4A + 4B = 0$ —①

$-4A + 4B = 6$ —②

Solving the equations using elimination method

From equ①

$4A = -4B$

$A = \frac{-4B}{4}$

$A = -B$

Sub $A=-B$ in equ ② therefore

$-4(-B)+4B = 6$

$4B + 4B = 6$

$8B = 6$

$B = \frac{6}{8}$

$B = 3/4$

Since $A = -B$

$A = -\frac{3}{4}$

P.I: $y = -3/4\cos\theta + 3/4\sin\theta$

G.S $y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4(\cos\theta - \sin\theta)$

(b) $PI = 0$

$y = -3/4\cos\theta + 3/4\sin\theta$

$-3/4\cos\theta + \frac{3}{4}\sin\theta = 0$

$-\frac{3}{4}\cos\theta = -3/4\sin\theta$

$-\cos\theta = -\sin\theta$

Dividing both sides by $-\cos\theta$

$\frac{-\cos\theta}{-\cos\theta} = \frac{-\sin\theta}{-\cos\theta}$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

② $EI\, d^2y/dx^2 = \frac{w}{2}(L-x)^2$

$EI.\, m^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$y = e^{0x}(A + Bx)$

$CF = y = A + Bx$

$P.I = y = Ex^2 + Fx^3 + Gx^4$

$\frac{dy}{dx} = 2Ex + 3Fx^2 + 4Gx^3$

$\frac{d^2y}{dx^2} = 2E + 6Fx + 12Gx$

$EI[2E + 6Fx + 12Gx^2] = w(L-x^2)$

$2EEI + 6EFIx + 12GEIx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4EEI + 12EFIx + 24GEIx^2 = w(L^2 - 2Lx + x^2)$

$24GEI = w$

$G = \frac{w}{24EI}$ .. ①

$12EFI = -2wL$

$F = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$ .. ②

$4EEI = wL^2$

$E = \frac{wL^2}{4EI}$

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$$y = \left[\frac{wl^2}{4EI}\right]x^2 - \left[\frac{wl}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$= \frac{wl^2x^2}{4EI} - \frac{wlx^3}{6EI} + \frac{wx^4}{24EI}$$

$$= \frac{6wl^2x^2 - 4wlx^2 + wx^4}{24EI}$$

$$= \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0, x = 0$ $dy/dx = 0$

$$0 = A + B(0) + w/24EI\left[6L^2(0) - 4L(0) + 0\right]$$

$$A = 0$$

$$dy/dx = B + \frac{w}{24EI}\left[6L^2(0) - 4L(0) + 0\right]$$

$$0 = B + \frac{w}{24EI}\left(12(0) - 12(0) + 4(0)\right)$$

$$B = 0$$

Particular solution

$$y = w/24EI\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{wx}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$y = \frac{wx^2}{24EI}\left(x^2 - 4Lx + 6L^2\right)$$

When $x = L$

$$y = \frac{wL^2}{24EI}\left(L^2 - 4L^2 + 6L^2\right)$$

$$y = \frac{wL^4}{8EI}$$