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15/ENG06/059
Mechanical Engineering.
ENG 381
Assignment.

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution Note: $\frac{dy}{dx} = m$

CF: $m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2 \& -1$

$y = Ae^{-x} + Be^{2x}$

PF

Assumed PI: $y = c$ ——— (i)

$\frac{dy}{dx} = 0$ ——— (ii)

$\frac{d^2y}{dx^2} = 0$ ——— (iii)

Subst. eqn (i), (ii) (iii) into the original equation.

$0 - 0 - 2(c) = 8$

$C = \frac{8}{-2} = -4$

$\therefore y = -4$

General solution is $y = CF + PI$

$y = Ae^{-x} + Be^{2x} - 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Solution Note $d^2y/dx = m$

CF: $m^2 - 4 = 0$

$(m+2)(m-2) = 0$

$m = -2 \; \& \; 2$

$y = Ae^{-2x} + Be^{2x}$

PF

Assumed PI $\quad y = Ce^{3x}$ ——— (i)

$\frac{dy}{dx} = 3Ce^{3x}$ ——— (ii)

$\frac{d^2y}{dx^2} = 9Ce^{3x}$ ——— (iii)

Subst. (i), (ii) & (iii) unto the original eqn

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = 2$

$\therefore$ PF : $y = 2e^{3x}$

The general soln is : CF + PF

$y = Ae^{-2x} + Be^{2x} + 2e^{3x}$.

(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

solution $\quad$ Note : $\frac{dy}{dx} = m$

CF: $m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

CF: $y = e^{-x}(A + Bx)$

PF

Assumed PI : $y = Ce^{-2x}$ ——— (i)

$\frac{dy}{dx} = -2Ce^{-2x}$ ——— (ii)

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$ ——— (iii) subst. eqn (i),(ii),(iii) into the orginal eqn

$4\cancel{ce^{-2x}} + 2(\cancel{-2}ce^{-2x}) + (ce^{-2x}) = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = 1$

PI : $y = e^{-2x}$

General solution : CF + PI

$y = e^{-x}(A + Bx) + e^{-2x}.$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

CF solution Note $dy/dx = m$

$m^2 + 25 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{0 \pm \sqrt{0^2 - 4(1)(25)}}{2a}$

$m \frac{0 \pm \sqrt{-100}}{2a} = \frac{0 \pm \sqrt{100} \times \sqrt{-1}}{2}$

$m = \frac{0 \pm 10j}{2} = 0 \pm 5j$

$y = e^0(A\cos 5x + B\sin 5x) = (A\cos 5x + B\sin 5x)$

PF

Assumed PI $y = Cx^2 + Dx + E + Cx + D$ —— (i)

$\frac{dy}{dx} = 2Cx + D + C$ —— (ii)

$\frac{d^2y}{dx^2} = 2C$ —— (iii)

Substitute eqn (i), (iii) in the original eqn

$25(Cx^2 + Dx + E + (x + D)) + 2C = 5x^2 + x$

$25[Cx^2 + (C + D)x + E + D] + 2C = 5x^2 + x$

$25Cx^2 + 25x(C + D) + 25E + 25D + 2C = 5x^2 + x$

Comparing Coefficient

for x^2

$25C = 5$

$C = 1/5$

for x

$25(C+D) = 1$

$C + D = 1/25$

$D = 1/25 - 1/5 = \frac{1-5}{25}$

$D = -4/25$

for the remaining

$25E + 25D + 2C = 0$

$25E + 25(-4/25) + 2(1/5) = 0$

$25E = 4 - 2/5 = \frac{20-2}{5} = 18/5$

$E = \frac{18}{5} \times \frac{1}{25} = \frac{18}{125}$

$\therefore$ PF: $y = 1/5\, x^2 - 4/25\, x + \frac{18}{125} + 1/5\, x + 4/25$

$y = 1/5\, x^2 + 1/25\, x - 2/125$

$y = 1/5\left(x^2 + 1/5\, x - 2/25\right)$

General soln : CF + PF

$y = \{A\cos 5x + B\sin 5x + 1/5\left(x^2 + 1/5\, x - 2/25\right)\}$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Solution note $\frac{dy}{dx} = m$

CF

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$

PF

Assumed PI $y = C\cos x + D\sin x$ —— (i)

$y' = -C\sin x + D\cos x$ —— (ii)

$y'' = -C\cos x - D\sin x$ —— (iii)

Substitute the eqns (i),(ii)&(iii) into the original eqn

$(-C\cos x - D\sin x) - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$\cancel{-C\cos x} - \cancel{D\sin x} + 2C\sin x - 2D\cos x + \cancel{C\cos x} + \cancel{D\sin x} = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing coefficient's

$2C = 4$

$C = 2$

$-2D = 0$

$D = 0$

PF $= y = 2\cos x$

General soln: $y = CF + PF$

$y = e^{x}(A + Bx) + 2\cos x$.

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$.

Soln Note! $dy/dx = m$

CF $m^2 + 4m + 5 = 0$

$\cancel{m^2 + 5m + 5 = 0}$ $\quad m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cancel{(m+5)(m-1) = 0}$

$\cancel{m = -5 \& 1}$ $\quad m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2} = \frac{-4 \pm \sqrt{16-20}}{2}$

$\cancel{y = Ae^{-5x} + Be^{x}}$

$m = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

PF

Assumed PI $y = Ce^{-2x}$ —— (i)

$\frac{dy}{dx} = -2Ce^{-2x}$ —— (ii)

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$ —— (iii)

Substitute eqn (i), (ii) & (iii) in the original Equation.

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$y = 2e^{-2x}$

General solution : $y = CF + PF$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

but when $x = 0$ & $y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = A + 2$

$A = -1$

but when $x = 0$ & $dy/dx = -2$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

$-2 = e^{0}(-A\sin 0 + B\cos 0) - 2e^{0}(A\cos 0 + B\sin 0) - 4e^{0}$

$-2 = B - 2A - 4$

$4 - 2 + 2A = B$

$2 + 2(-1) = B$

$B = 0$

∴ The general soln:

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

(7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$

soln & Note: $dy/dx = m$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$(3m - 3)(m + 1) = 0$

$m = 1$ & -1

$y = Ae^{-x} + Be^{x}$

~~Assumed~~ PE

Assumed PI : $y = Cx + D$ —— (i)

$\frac{dy}{dx} = C$ —— (ii)

$\frac{d^2y}{dx^2} = 0$ —— (iii)

Substitute ~~into~~ eqn (i)(ii) & (iii) into the original eqn

$3(0) - 2C - Cx - D = 2x - 3$

$-2C - Cx - D = 2x - 3$

Comparing coefficient

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$4 + 3 = D$

$D = 7$

PF : $y = -2x + 7$

General soln : $y = PF + CF$

$y = Ae^{-x} + Be^{x} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln : Note $\frac{dy}{dx} = m$

CF $m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$(m-4)(m-2) = 0$

$m = 4$ & 2

$y = Ae^{2x} + Be^{4x}$

PF

Assumed PI $y = Ce^{4x}$ ~~(i)~~ $= Cxe^{4x}$ — (i)

$\frac{dy}{dx} = 4Ce^{4x}$ — (ii) $\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$ — (ii)

$\frac{d^2y}{dx^2} = 16Ce^{4x}$ — (iii) $\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$d^2y/dx^2 = 16Cxe^{4x} + 8Ce^{4x}$

Substitute eqn (i), (ii) & (iii) into the original eqn

$$16(Ce^{4x}) - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

$$16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$$

$$\cancel{16Cxe^{4x}} + 8Ce^{4x} - \cancel{24Cxe^{4x}} - 6Ce^{4x} + \cancel{8Cxe^{4x}} = 8e^{4x}$$

$$2Ce^{4x} = 8e^{4x}$$

comparing coefficient

$$2C = 8$$

$$C = 4$$

$\therefore$ PF: $y = 4xe^{4x}$

The general soln:

$$y = Ae^{2x} + Be^{4x} + 4xe^{4x}$$