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MATRIC NO: 16/ENG07/027

DEPT: PETROLEUM ENG.

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

soln

$y'' - y' - 2y = 0$

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$

$(k^2 - k - 2)e^{kx} = 0$

$k^2 - k - 2 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-2)}}{2 \times 1}$

$= \frac{1 \pm \sqrt{1 - (-8)}}{2}$

$= \frac{1 \pm \sqrt{9}}{2}$

$= \frac{1 \pm 3}{2} = \frac{1+3}{2}, \frac{1-3}{2}$; $k_1 = 2$ $k_2 = -1$

$y_1 = e^{k_1 x} = e^{2x}$, $y_2 = e^{k_2 x} = e^{-x}$

$y = c_1y_1 + c_2y_2$

$y_n = c_1e^{2x} + c_2e^{-x}$

$y_p = c$, $y'_p = 0$, $y''_p = 0$

$0 - 0 - 2(c) = 8$

$\frac{-2}{2}c = \frac{8}{2}$

$c = -4$

$y = c_1e^{2x} + c_2e^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Soln

$y'' - 4y = 0$

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} - 4e^{kx} = 0$

$(k^2 - 4)e^{kx} = 0$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \pm\sqrt{4}$

$k = \pm 2$; $k_1 = 2$, $k_2 = -2$

$y_1 = e^{k_1x} = e^{2x}$, $y_2 = e^{k_2x} = e^{-2x}$

$y_h = c_1e^{2x} + c_2e^{-2x}$

$y_h = c_1\cosh 2x + c_2\sinh 2x$

~~$y_p = ce^{3x}$~~

$y'_p = 3ce^{3x}$

$y''_p = 9ce^{3x}$

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$5ce^{3x} = 10e^{3x}$

$\frac{5c}{5} = \frac{10}{5}$

$c = 2$

$y_p = ce^{3x}$

$y_p = 2e^{3x}$

$y = c_1\cosh 2x + c_2\sinh 2x + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

soln

$y'' + 2y' + y = 0$

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} + 2(ke^{kx}) + e^{kx} = 0$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$(k^2 + 2k + 1)e^{kx} = 0$

$k^2 + 2k + 1 = 0$

k^2

$k \quad k$

$k^2 + k + k + 1 = 0$

$k(k+1) + 1(k+1) = 0$

$k+1 = 0 \qquad k+1 = 0$

$k_1 = -1 \qquad k_2 = -1$

$y_1 = e^{k_1x} = e^{-x}$, $y_2 = e^{k_2x} = e^{-x}$

$y = c_1e^{-x} + xc_2e^{-x}$

$y_h = (c_1 + xc_2)e^{-x}$

$y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$(4A - 4A + A)e^{-2x} = e^{-2x}$

$A = 1$

$y_p = Ae^{-2x}$

$y_p = e^{-2x}$

$y = (c_1 + xc_2)e^{-x} + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Soln

$y'' + 25y = 0$

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} + 25(e^{kx}) = 0$

$k^2 e^{kx} + 25e^{kx} = 0$

$(k^2 + 25) e^{kx} = 0$

$k^2 + 25 = 0$

$k^2 = 25$

$k = \pm\sqrt{25}$

$k_1 = 5, \quad k_2 = -5$

$y_1 = e^{k_1 x} = e^{5x}, \quad y_2 = e^{k_2 x} = e^{-5x}$

$y_h = c_1 e^{5x} + c_2 e^{-5x}$

$y_h = c_1 \cosh 5x + c_2 \sinh 5x$

$y_p = 5x^2 + x$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$(2A + 25C) + 25Ax^2 + 25Bx$

$x: \; 25B = 1$

$B = 1/25$

$x^2: \; 25A = 5$

$A = 5/25 = 1/5$

$2A + 25C = 0$

$2(1/5) + 25C = 0$

$2/5 + 25C = 0$

$\frac{2/5}{-25} = \frac{-25C}{-25}$

$C = -2/125$

$y = c_1 \cosh 5x + c_2 \sinh 5x + 1/5 x^2 + 1/25 x - 2/125$

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5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Soln

$y'' - 2y' + y = 0$

$k^2e^{kx} - 2(ke^{kx}) + e^{kx} = 0$

$(k^2 - 2k + 1)e^{kx} = 0$

$k^2 - 2k + 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times 1}}{2 \times 1}$

$\frac{2 \pm \sqrt{4-4}}{2}$

$k_1 = 1$ & $k_2 = 1$

$y_1 = e^{k_1 x} = e^x$, $y_2 = e^{k_2 x} = e^x$

$y_h = C_1e^x + xC_2e^x$

$y_h = (C_1 + xC_2)e^x$

$y_p = A\cos x + B\sin x$

$y'_p = -A\sin x + B\cos x$

$y''_p = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2(-A\sin x + B\cos x) + A\cos x + B\sin x = 4\sin x$

$-A\cos x - B\sin x + 2A\sin x - 2B\cos x + A\cos x + B\sin x = 4\sin x$

$(2A)\sin x + (-2B)\cos x = (4)\sin x + (0)\cos x$

$2A = 4$, $-2B = 0$

$A = 4/2 = 2$, $B = 0$

$y_p = A\sin x + B\cos x$

$y_p = 2\cos x + 0\sin x = 2\cos x$

$y = y_h + y_p$

$y = (c_1 + x c_2)e^{2x} + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Given that at $x=0$ $y=1$ & $dy/dx = -2$

Soln

$y'' + 4y' + 5y = 2e^{-2x}$

$y'' + 4y' + 5y = 0$

$k^2e^{kx} + 4(ke^{kx}) + 5(e^{kx}) = 0$

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$(k^2 + 4k + 5)e^{kx} = 0$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + 4 = -5 + 4$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$k_1 = -2 + i$

$k_2 = -2 - i$

$y_h = c_1e^{(-2+i)x} + c_2e^{(-2-i)x}$

$y_h = c_1e^{-2x}\cdot e^{ix} + c_2e^{-2x}\cdot e^{-ix}$

$y_h = [c_1e^{ix} + c_2e^{-ix}]e^{-2x}$

$y_h = e^{-2x}[c_1\cos x + c_2\sin x]$

$y_h = e^{-2x}[A\cos x + B\sin x]$

$y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$A = 2$

$y_p = 2e^{-2x}$

$y = e^{-2x}[A\cos x + B\sin x] + 2e^{-2x}$

$1 = e^{-2(0)}[A\cos(0) + B\sin(0)] + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2 = -1$

$y' = [-2e^{-2x}(A\cos x + B\sin x)] + [e^{-2x}(-A\sin x + B\cos x)] - 2e^{-2x}$

$y' = -2Ae^{-2x}\cos x - 2Be^{-2x}\sin x - Ae^{-2x}\sin x + Be^{-2x}\cos x - 4e^{-2x}$

$-2 = -2Ae^{-2(0)}\cos(0) - 2Be^{-2(0)}\sin(0) - Ae^{-2(0)}\sin(0) + Be^{-2(0)}\cos(0) - 4e^{-2(0)}$

$-2 = -2A + B - 4$

$B = -2 + 2A + 4$

$B = -2 + 2(-1) + 4$

$B = -2 - 2 + 4$

$B = 0$

$y = e^{-2x}[-\cos x + 0\sin x] + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}[2 - \cos x]$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Solution

$3y'' - 2y' - y = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k) + (k - 1) = 0$

$3k(k-1) + 1(k-1) = 0$

$(k-1)(3k+1) = 0$

$k_1 = 1 \quad \& \quad k_2 = -1/3$

$y_h = c_1e^x + c_2e^{-x/3}$

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

$3(0) - 2(A) - (Ax + B) = 2x - 3$

$-2A - Ax - B = 2x - 3$

$(-2A - B) - Ax = 2x - 3$

$x: -Ax = 2x$

$A = -2$

$-2A - B = -3$

$-2(-2) - B = -3$

$B = 4 + 3 = 7$

$y_2 = -2x + 7$

$y = C_1 e^{x} + C_2 e^{-x/3} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln

$y'' - 6y' + 8y = 8e^{4x}$

$k^2 - 6k + 8 = 0$

$(k^2 - 4k)(-2k + 8) = 0$

$k(k-4) - 2(k-4) = 0$

$(k-4)(k-2) = 0$

$k_1 = 4, \quad k_2 = 2$

$y_h = C_1 e^{4x} + C_2 e^{2x}$

$y_p = Axe^{4x}$

$y'_p = A[e^{4x} + 4xe^{4x}] = Ae^{4x} + 4Axe^{4x}$

$y''_p = 4Ae^{4x} + 4A(e^{4x} + 4xe^{4x})$

$y''_p = 4Ae^{4x} + 4Ae^{4x} + 16Axe^{4x} = 8Ae^{4x} + 16Axe^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6(Ae^{4x} + 4Axe^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6Ae^{4x} - 24Axe^{4x} + 8Axe^{4x} = 8e^{4x}$

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8) $2Ae^{4x} = 8e^{4x}$

$2A = 8$

$A = 4$

$y_p = 4xe^{4x}$

$y = C_1e^{4x} + C_2e^{2x} + 4xe^{4x}$