

$$2) EI \frac{d^2y}{dx^2} = \frac{w}{2} (L-x)^2$$

C.F.

Auxiliary Equation

$$m^2 = 0$$

$$m = \sqrt{0}$$

$$m = \frac{1}{2} \cdot 0 \text{ twice}$$

$$y = e^0 (A + Bx)$$

$$y = A + Bx$$

Assume P.I.

$$y = Cx^2 + Dx^3 + fx^4 \quad \text{--- (1)}$$

$$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4fx^3 \quad \text{--- (2)}$$

$$\frac{d^2y}{dx^2} = 2C + 6Dx + 12fx^2 \quad \text{--- (3)}$$

∴ Cutting eqn 3 into the Original Equation

$$EI (2C + 6Dx + 12fx^2) = \frac{w}{2} (L-x)^2$$

$$2CEI + 6DEIx + 12EFIx^2 = \frac{w}{2} (L-x)^2$$

$$2CEI + 6DEIx + 12EFIx^2 = \frac{w}{2} (L-x)(L-x)$$

$$2CEI + 6DEIx + 12EFIx^2 = \frac{w}{2} (L^2 - 2Lx + x^2)$$

$$4CEI + 12DEIx + 24EFIx^2 = w(L^2 - 2Lx + x^2)$$

$$4CEI + 12DEIx + 24EFIx^2 = w(L^2 - 2Lx + x^2)$$

Comparing Co-efficients

$$x^2: 24EFI = 2w$$

$$f = \frac{w}{24EI}$$

$$x: 12DEI = -2wL$$

$$D = \frac{-2wL}{12EI}$$

$$\text{Constant: } 4CEI = wL^2$$

$$C = \frac{wL^2}{4EI}$$

Putting back into the Original equation (P.I)

$$y = \frac{wL^2}{4EI} x^2 + \frac{-2wL}{12EI} x + \frac{w}{24EI} x^4$$

∴ The general solution

$$y = C.F. + P.I$$

$$= A + Bx + \frac{wL^2}{4EI} x^2 - \frac{2wL}{12EI} x + \frac{w}{24EI} x^4$$

$$\frac{dy}{dx} = B + \frac{2wL^2x}{4EI} - \frac{6wLx}{12EI} + \frac{4wx^3}{24EI}$$

at $y=0$ and $x=0$

$$y = A + Bx + \frac{wL^2}{4EI} x^2 - \frac{2wL}{12EI} x + \frac{w}{24EI} x^4$$

$$0 = A$$

$$A = 0$$

at $\frac{dy}{dx} = 0$ and $x=L$

$$0 = B + \frac{2wL^2x}{4EI} - \frac{6wLx}{12EI} + \frac{4w}{24EI} x^3$$

$$0 = B + \frac{wL^2}{2EI} - \frac{wL^2x}{2EI} + \frac{w}{3EI}$$

$$B = 0$$

General equation with k

$$y = \frac{wL^2}{4EI} x^2 - \frac{2wL}{12EI} x + \frac{w}{24EI} x^4$$

$$y = \frac{wL^2}{4EI} x^2 - \frac{wL}{6EI} x + \frac{w}{24EI} x^4$$

at $x=L$

$$y = \frac{wL^2}{4EI} - \frac{wL^2}{6EI} + \frac{wL^4}{24EI}$$

$$y = \frac{6wL^4}{24EI} - \frac{4wL^4}{24EI} + \frac{wL^4}{24EI}$$

$$y = \frac{3wL^4}{24EI}$$

$$y = \frac{wL^4}{8EI}$$

$$1.) \frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 5y = 6 \sin \theta$$

C.F

$$m^2 + 4m + 5 = 0$$

This is a complex equation

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{4^2 - 20}}{2(1)}$$

$$\frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$\frac{-4 \pm \sqrt{-4}}{2}$$

$$\frac{-4 \pm 2j}{2}$$

$$= -2 \pm j$$

$$\therefore y = e^{-2\theta} (A \cos \theta + B \sin \theta)$$

Assumed P.I.

$$y = D \cos \theta + D \sin \theta$$

$$\frac{dy}{d\theta} = -D \sin \theta + D \cos \theta$$

$$\frac{d^2y}{d\theta^2} = -D \cos \theta - D \sin \theta$$

$$\frac{d^2y}{d\theta^2} = -D \cos \theta - D \sin \theta$$

$$-D \cos \theta - D \sin \theta + 4(-D \sin \theta + D \cos \theta) +$$

$$5(D \cos \theta + D \sin \theta) = 6 \sin \theta$$

$$-D \cos \theta - D \sin \theta - 4D \sin \theta + 4D \cos \theta +$$

$$5D \cos \theta + 5D \sin \theta = 6 \sin \theta$$

Comparing Co-efficients

$$\sin \theta: -D - 4D + 5D = 6$$

$$\cos \theta: -D + 4D + 5D = 0$$

$$4D - 4D = 6 \quad \text{--- (1)}$$

$$4D + 4D = 0 \quad \text{--- (2)}$$

$$4D - 4D = 6$$

$$4D + 4D = 0$$

$$-8D = 6$$

$$D = -6/8$$

$$C = -3/4$$

from eqn ii

$$4(-3/4) + 4D = 6$$

$$-3 = 4D$$

$$D = 3/4$$

\(\therefore\) Assumed P.I

$$\therefore y = -3/4 \cos \theta + 3/4 \sin \theta$$

$$y = 3/4 \sin \theta - 3/4 \cos \theta$$

General Equation

$$= C.F + P.I$$

$$y = e^{-2\theta} (A \cos \theta + B \sin \theta) + 3/4 \sin \theta - 3/4 \cos \theta$$

$$ii) y = 3/4 (\sin \theta - \cos \theta)$$

for \(\theta = 0\) to \(2\pi\)

Considering P.I

$$y = 3/4 (\sin \theta - 3/4 \cos \theta)$$

$$\frac{dy}{d\theta} = 3/4 \cos \theta + 3/4 \sin \theta$$

at Steady State:

$$\frac{dy}{d\theta} = 0 \text{ and } \theta = \text{infinity as}$$

$$0 = 3/4 (\cos \theta + \sin \theta)$$

$$-\cos \theta = \sin \theta$$

divide through by \(\cos \theta\)

$$-\frac{\cos \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

$$\theta = 315^\circ$$