$\therefore x = 1$ (for absolute Convergence)

Case 1: For $|x| < 1$, Put $x = -1$

$$U_n = \frac{(-1)^n}{(2n+1)^n} < \frac{1}{n^2}$$

$$= \frac{1}{(2n+1)^3} > \frac{1}{n^3} \text{ for } P = 3$$

$x = -1$ (for absolute Convergence)

$\therefore$ The range of Values for x for which the series U_n is absolutely Convergent is:

$$-1 \le x \le 1$$

4) Evaluate Using L'Hopital Rule

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) = \frac{\cos x + \sin x}{3x^2}$$

$$= \frac{-\sin x + \cos x}{6x} = \frac{-\cos x - \sin x}{6}$$

when $x \to 0$ $\quad \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6} \quad \frac{-1}{6}$

b) Given $\lim_{x \to \frac{\pi}{2}} \ln\left(\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right)$

Note $\ln e^{(x)} = (x)$

$\Rightarrow \left(\frac{3x^2+2x-1}{x+1}\right)$

$\frac{dy}{dx} = \frac{6x+2}{1}$

when $x \to \frac{\pi}{2}$ $\frac{6(\frac{\pi}{2})+2}{1} = 3\pi + 2$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$\cos\left[\sin^{-1}\frac{(2+\sqrt{3})-2}{((2+\sqrt{3})-\sqrt{3})}\right]$

$\cos\left[\sin^{-1}\frac{\sqrt{3}}{2}\right]$

when $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\cos\left(\frac{\pi}{3}\right) = 0.5$ or $\frac{1}{2}$

$\therefore \lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$

d) $\lim \left[x^2 - 8x + 1 \ldots \right.$

Scanned by CamScanner

$\therefore x = 1$ (for absolute Convergence)

Case 1: For $|x| < 1$, Put $x = -1$

$$U_n = \frac{(-1)^n}{(2n+1)^n} < \frac{1}{n^2}$$

$$= \frac{1}{(2n+1)^3} > \frac{1}{n^3} \text{ for } P = 3$$

$x = -1$ (for absolute Convergence)

$\therefore$ The range of Values for x for which the Series U_n is absolutely Convergen is:

$$-1 \leq x \leq 1$$

4) Evaluate Using L'Hospital Rule

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) = \frac{\cos x + \sin x}{3x^2}$$

$$= \frac{-\sin x + \cos x}{6x} = \frac{-\cos x - \sin x}{6}$$

when $x \to 0$ $\quad \frac{-\cos 0 - \sin 0}{6} = \frac{-1 - 0}{6} \quad \frac{-1}{6}$

Scanned by CamScanner

2c) $U_n = \frac{1+2n^2}{1+n^2}$

$U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2} = \frac{2n^2+4n+3}{n^2+2n+2}$

$\lim_{n\to\infty} \left|\frac{U_{n+1}}{U_n}\right| = \frac{2n^2+4n+3}{n^2+2n+2} \div \frac{1+n^2}{1+2n^2}$

Divide through by n^2

$\lim_{n\to\infty} = \frac{\left(2+\frac{4}{n}+\frac{3}{n^2}\right)\cdot\left(\frac{1}{n^2}+1\right)}{\left(1+\frac{2}{n}+\frac{2}{n^2}\right)\cdot\left(\frac{1}{n^2}+2\right)} = \frac{2}{2} = 1$

$\therefore U_n = \frac{1+2n^2}{1+n^2}$ is inconclusive or Undefined Convergence

Comparison with the p-Series

$U_n = \frac{1+2n^2}{1+n^2}$ p-Series $= \frac{1}{n^2}$ for $p=2$

For $n=1$

$U_n = \frac{3}{2} > 1$

for $n=2$

$\frac{9}{5} = \frac{1}{4}$

From the above test,

$U_n > \frac{1}{n^2}$

i.e. $\frac{1+2n^2}{1+n^2} > \frac{1}{n^2}$ for $p=2$

$\therefore$ The Series $U_n = \frac{1+2n^2}{1+n^2}$ diverges It does not Converge

$\frac{}{2\times3} + \frac{}{3\times4} + \frac{}{4\times5} + \frac{}{5\times6} + \cdots$

$$U_n = \frac{x}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{x}{(n+2)(n+3)}$$

$$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \left|\frac{x}{(n+1)(n+2)} \times \frac{(n+2)(n+1)}{x}\right| = \left|\frac{n+1}{n+3}\right|$$

$$\lim_{n\to\infty} = \left|\frac{1+\frac{1}{n}}{1+\frac{3}{n}}\right| = \frac{1}{1} = 1$$

$\therefore$ Since $\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = 1$, U_n inconclusive or Undetermined Convergence

Use P-Series to Compare with the Series

$U_n = \frac{x}{(n+1)(n+2)}$ P-Series $= \frac{1}{n^p}$

For $P=2$

$$\frac{x}{(n+1)(n+2)} > \frac{1}{n^2}$$

Test For $n = 1, 2, 3, 4$

Note $x = 2$ for $U_n = \frac{x}{(n+1)(n+2)}$

$$U_n = \frac{2}{(n+1)(n+2)}$$

For $n=1$

$$\frac{2}{2\times3} < \frac{1}{1}$$

For $n=2$

$$\frac{2}{2\times4} < \frac{1}{4}$$

2c) $U_n = \frac{1+2n^2}{1+n^2}$

$$U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2} = \frac{2n^2+4n+3}{n^2+2n+2}$$

$$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \frac{2n^2+4n+3}{n^2+2n+2} \div \frac{1+n^2}{1+2n^2}$$

Divide through by n^2

$$\lim_{n\to\infty} = \frac{\left(2+\frac{4}{n}+\frac{3}{n^2}\right)\cdot\left(\frac{1}{n^2}+1\right)}{\left(1+\frac{2}{n}+\frac{2}{n^2}\right)\cdot\left(\frac{1}{n^2}+2\right)} = \frac{2}{2} = 1$$

$\therefore U_n = \frac{1+2n^2}{1+n^2}$ is inconclusive or undefined convergence.

Comparison with the p-series

$U_n = \frac{1+2n^2}{1+n^2}$ p-series $= \frac{1}{n^2}$ for $p=2$

For $n=1$

$U_n = \frac{3}{2} > 1$

For $n = 2$

$\frac{9}{5} = \frac{1}{4}$

From the above test,

$U_n > \frac{1}{n^2}$

i.e. $\frac{1+2n^2}{1+n^2} > \frac{1}{n^2}$ for $p=2$

$\therefore$ The series $U_n = \frac{1+2n^2}{1+n^2}$ diverges (it doesn't converge)

$\frac{}{2\times3}+\frac{}{3\times4}+\frac{}{4\times5}+\frac{}{5\times6}+$

$U_n = \frac{x}{(n+1)(n+2)}$

$U_{n+1} = \frac{x}{(n+2)(n+3)}$

$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \left|\frac{x}{(n+1)(n+2)} \times \frac{(n+2)(n+1)}{x}\right| = \left|\frac{n+1}{n+3}\right|$

$\lim_{n\to\infty} = \left|\frac{1+\frac{1}{n}}{1+\frac{3}{n}}\right| = \frac{1}{1} = 1$

$\therefore$ Since $\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = 1$, U_n inconclusive or Undetermined Convergence

Use P-Series to Compare with the Series

$U_n = \frac{x}{(n+1)(n+2)}$ P-Series $= \frac{1}{n^p}$

For $P=2$

$\frac{x}{(n+1)(n+2)} > \frac{1}{n^2}$

Test For $n=1,2,3,4$

Note $x=2$ for $U_n = \frac{x}{(n+1)(n+2)}$

$U_n = \frac{2}{(n+1)(n+2)}$

For $n=1$

$\frac{2}{2\times3} < \frac{1}{1}$

For $n=2$

$\frac{2}{2\times4} < \frac{1}{4}$

Arpon Nsikan Paul
Mechatronics ENG
16/ENG05/007
ENG 281

2) Find the range of Values for x for whic the Series below is absolutely Convergent

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

Sol

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\left|\frac{U_{n+1}}{U_n}\right| = \left|\frac{x^{n+1}}{(2n+3)^3} \cdot \frac{(2n+1)^3}{x^n}\right| = \left|\frac{x^n \cdot x^1}{(2n+3)^3} \cdot \frac{(2n+1)^3}{x^n}\right|$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

divid both Numerator and denominator by n

$$= \frac{x(2+\frac{1}{n})^3}{(2+3/n)^3}$$

$\lim_{x\to\infty} \left|\frac{U_{n+1}}{U_n}\right| = x$, for absolute Convergence $\lim_{x\to\infty} \left|\frac{U_{n+1}}{U_n}\right| < 1$

For Convergence $|x| < 1$

$$\Rightarrow \frac{x\cancel{(2)^3}}{\cancel{8}^3} \Rightarrow \cdots -1 < x < 1$$

$\therefore$ Therefor the Series $U_n < \frac{1}{n^p}$ for only $n \leq 3$

$\therefore$ Therefore, for the Series $U_n = \frac{x}{(n+1)(n+2)}$ It can not be Said to be a Converging Series; It may be divergent, the Series is inconclusive convergence.

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2}$

$\therefore U_n = \frac{x}{n^2}, \quad U_{n+1} = \frac{x}{(n+1)^2}$

$$\lim_{n\to\infty} \left| \frac{U_{n+1}}{U_n} \right| = \frac{x}{n^2+2n+1} \times \frac{n^2}{x} \therefore = \frac{n^2}{n^2+2n+1}$$

Dividing through by n^2

$$\frac{U_{n+1}}{U_n} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = 1$$ (inconclusive or Undetermined

Comparison With the P - Series

$U_n = \frac{2}{n^2}$ and $\frac{1}{n^2}$ For $P = 2$

Since $x = 2$ For U_n

$U_n > \frac{1}{n^p}$ (i.e $\frac{2}{n^2} > \frac{1}{n^2}$)

Therefore the Series $U_n = \frac{x}{n^2}$ does not converge but diverges.

1) Evaluate the following limits of function

a) $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4})\sin(\cos x)}{x - \frac{\pi}{2}} \right]$

Sol

$\frac{U}{V}$, look for $\frac{U'}{V'}$

for $U = x^2 \sin(\cos x) - \frac{\pi}{4} \sin(\cos x)$

$w = \cos x$, $y = \sin(w)$

$\frac{dw}{dx} = -\sin x$, $\frac{dy}{dx}$ $\cos(w)$

$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx} = \cos(w) \cdot (-\sin x)$

$\frac{dy}{dx} = \cos(\cos x)(-\sin x)$

$= \cancel{[-x^2 \cos(\cos x)(\sin x)}$

Using Product rule

$a = x^2$

$b = \sin(\cos x)$

$= [-x^2 \cos(\cos x)(\sin x) + \sin(\cos x)2x] + \frac{\pi}{4} \cos(\cos x)(\sin x)$