

PAUL

15/ENGG03/023

Civil Engineering

ENG 381

$$1 \quad \frac{d^2 y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$$

$$\text{let } 6\sin\theta = 0$$

$$m^2 + 4m + 5 = 0$$

$$a=1, b=4, c=5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm j\sqrt{4}}{2}$$

$$= \frac{-4 \pm j2}{2}$$

$$m = -2 + j$$

$$m = \alpha + j\beta$$

$$CF = y = e^{-2\theta} (A\cos\theta + B\sin\theta)$$

$$FC(\theta) = 6\sin\theta$$

$$P \cdot J = y = C\cos\theta + D\sin\theta$$

$$\frac{dy}{d\theta} = -\sin\theta + D\cos\theta$$

$$\frac{d^2 y}{d\theta^2} = -\cos\theta - D\sin\theta$$

$$\frac{d^2 y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$$

$$-(\cos\theta - D\sin\theta + 4C - C\sin\theta + 5C\cos\theta + 5D\sin\theta) = 6\sin\theta$$

$$4C \cos \theta + 4D \sin \theta - 4C \sin \theta + 4D \cos \theta = 6 \sin \theta$$

$$4C(\cos \theta + 4D \cos \theta + 4D \sin \theta - 4C \sin \theta) = 6 \sin \theta$$

$$(4C + 4D) \cos \theta + (4D - 4C) \sin \theta = 6 \sin \theta$$

$$4C + 4D = 0 \quad \dots (i)$$

$$-4C + 4D = 6 \quad \dots (ii)$$

$$8D = 6$$

$$D = \frac{3}{4}$$

From eqn (i)

$$4C + 4\left(\frac{3}{4}\right) = 0$$

$$4C + 3 = 0$$

$$4C = -3$$

$$C = \frac{-3}{4}$$

$$\text{P.I. : } y = \frac{-3}{4} \cos \theta + \frac{3}{4} \sin \theta$$

$$y_{\text{os}} = \text{C.F.} + \text{P.I.}$$

$$y = e^{-2x} (A \cos \theta + B \sin \theta) - \frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta$$

Steady State Equation

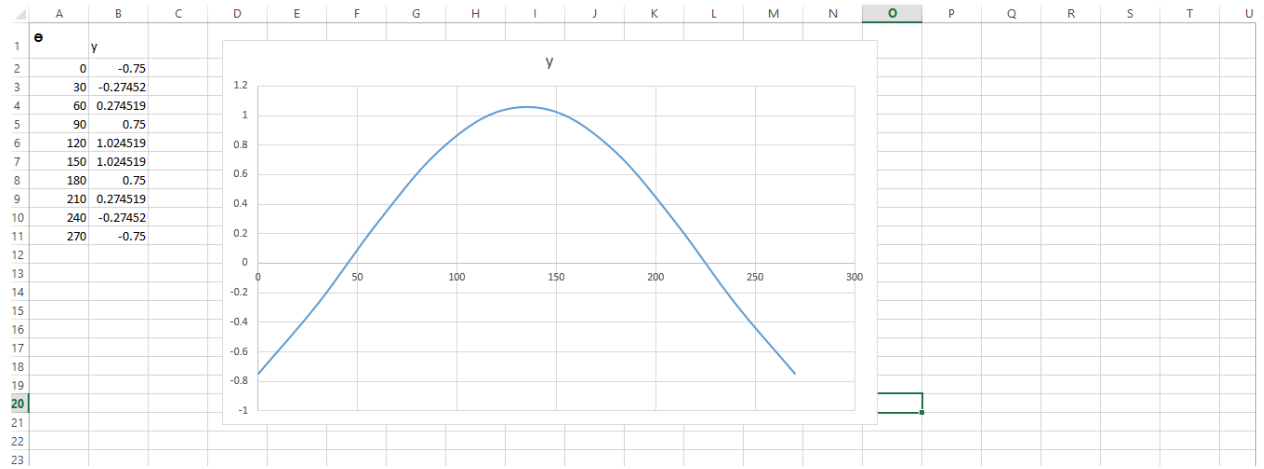
$$y' = +\frac{3}{4} \sin \theta + \frac{3}{4} \cos \theta = 0$$

$$\frac{\frac{3}{4} \sin \theta}{\frac{3}{4} \cos \theta} = \frac{-\frac{3}{4} \cos \theta}{\frac{3}{4} \cos \theta}$$

$$\tan \theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$



$$2 \quad EI \frac{d^2 y}{dx^2} = \frac{w}{2} (1-x)^2$$

$$EI m^2 = 0$$

$$m^2 = 0$$

$$m = +\sqrt{0}$$

$$m = \pm 0$$

$$y = e^{0x} (A + Bx)$$

$$C.F. = y = A + Bx$$

$$P.I = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$$

$$\frac{d^2 y}{dx^2} = 2F + 6Gx + 12Hx^2$$

$$EI [2F + 6Gx + 12Hx^2] = \frac{w}{2} (1-x)^2$$

$$2FEI + 6Gx EI + 12HEI x^2 = \frac{w}{2} (1-x)^2$$

$$4FEI + 12GEI + 24HEI x^2 = w(1^2 - 2Lx + x^2)$$

$$24HEI = w$$

$$H = \frac{w}{24EI} \quad \dots (1)$$

$$24FEI = -2wL$$

$$12GEI = -2wL$$

$$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \quad \dots (2)$$

$$4FEI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

$$4FEI$$

$$y = \left[\frac{wL^2}{4EI} \right] x^2 - \left[\frac{wL}{6EI} \right] x^3 + \left[\frac{w}{24EI} \right] x^4$$

$$= \frac{wL^2 x^2}{4EI} - \frac{wL x^3}{6EI} + \frac{w x^4}{24EI}$$

$$y = \frac{6\omega L^2 x^2 - 4\omega L x^3 + \omega x^4}{24EI}$$

$$G.S = y = A + Bx + \frac{\omega}{24EI} [6L^2 x^2 - 4Lx^3 + x^4]$$

$$y=0, \quad x=0 \quad \frac{dy}{dx} = 0$$

chc

$$0 = A + B(0) + \frac{\omega}{24EI} [6L^2(0) - 4L(0) + (0)]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{\omega}{24EI} [12L^2 x - 12Lx^2 + 4x^3]$$

chc

$$0 = B + \frac{\omega}{24EI} [12(0) - 12(0) + 4(0)]$$

$$B = 0$$

$$y = \frac{\omega}{24EI} [6L^2 x^2 - 4Lx^3 + x^4]$$

$$y = \frac{\omega x^2}{24EI} [6L^2 - 4Lx + x^2]$$

$$y = \frac{\omega x^2}{24EI} [x^2 - 4Lx + 6L^2]$$

When $x=L$

$$y = \frac{\omega L^2}{24EI} [L^2 - 4L^2 + 6L^2]$$

$$y = \frac{\omega L^2}{24EI} [3L^2]$$

$$y = \frac{\omega L^4}{8EI}$$