

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$$

Note: It is a non-homogeneous equation.

General Solution = Complementary Function + Particular integral

Complementary Function

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

Auxiliary equation

$$m^2 + 4m + 5 = 0$$

$$m^2 + 4m = -5$$

$$m^2 + 4m + (2)^2 = -5 + (2)^2$$

$$(m+2)^2 = -1$$

$$m+2 = \pm\sqrt{-1}$$

$$m+2 = \pm i$$

$$\therefore m = -2+i \quad \& \quad m = -2-i$$

$$\therefore CF = C_1 e^{(-2+i)\theta} + C_2 e^{(-2-i)\theta}$$

$$y_h = C_1 e^{-2\theta+i\theta} + C_2 e^{-2\theta-i\theta}$$

$$y_h = C_1 e^{-2\theta} e^{i\theta} + C_2 e^{-2\theta} e^{-i\theta}$$

$$y_h = e^{-2\theta} [C_1 e^{i\theta} + C_2 e^{-i\theta}]$$

$$y_h = e^{-2\theta} [A \cos\theta + B \sin\theta]$$

$$y_p = A \cos\theta + B \sin\theta$$

$$y_p' = -A \sin\theta + B \cos\theta$$

$$y_p'' = -A \cos\theta - B \sin\theta$$

$$-A \cos\theta - B \sin\theta + 4[-A \sin\theta + B \cos\theta] + 5A \cos\theta + 5B \sin\theta = 6 \sin\theta$$

$$-A \cos\theta - B \sin\theta - 4A \sin\theta + 4B \cos\theta + 5A \cos\theta + 5B \sin\theta = 6 \sin\theta$$

$$4A \cos\theta + 4B \sin\theta - 4A \sin\theta + 4B \cos\theta = 6 \sin\theta$$

$$(4A+4B) \sin\theta + (4A+4B) \cos\theta = 6 \sin\theta$$

$$-4A+4B = 6$$

$$4A+4B = 0$$

$$8B = 6$$

$$\therefore B = \frac{6}{8}$$

$$B = \frac{3}{4}$$

$$y = 0$$

$$y' = 0$$

$$y = \frac{w}{2}$$

$$y = \frac{w}{2} \cos \theta + \frac{w}{2} \sin \theta$$

$$y = \frac{w}{2} \sin \theta$$

$$y = \frac{w}{2} e^{-\theta} [A \cos \theta + B \sin \theta] + \frac{w}{2} \sin \theta - \frac{w}{2} \cos \theta$$

Apply this operation

$$y'' = 0$$

$$y'' = \frac{w}{2} \cos \theta + \frac{w}{2} \sin \theta = 0$$

$$\frac{w}{2} \cos \theta + \frac{w}{2} \sin \theta = 0$$

$$\frac{w}{2} \cos \theta = -\frac{w}{2} \sin \theta$$

$$\cos \theta = -\sin \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{-\cos \theta}{\cos \theta}$$

$$\tan \theta = -1$$

$$\therefore \tan \theta = 1$$

$$2) EI \frac{d^2 y}{dx^2} = \frac{w}{2} (L-x)^2$$

The auxiliary equation becomes

$$EI m^2 = 0$$

$$m^2 = 0$$

$$m = \pm \sqrt{0}$$

$$m = 0$$

$$\therefore y = e^{0x} [A + Bx]$$

$$y = A + Bx$$

$$y_p = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$$

$$\frac{d^2 y}{dx^2} = 2F + 6Gx + 12Hx^2$$

$$EI [2F + 6Gx + 12Hx^2] = \frac{w}{2} (L-x)^2$$

$$2FEI + 6GxEI + 12Hx^2EI = \frac{w}{2} (L-x)^2$$

$$4FEI + 12GxEI + 24Hx^2EI = w(L-x)^2$$

$$-12EI + 12EI + 24EI + w(L^2 - 2Lx + x^2)$$

$$24EI + w$$

$$H = \frac{w}{24EI}$$

$$4EI$$

$$12EI = 2wL$$

$$G = \frac{2wL}{12EI} = \frac{wL}{6EI}$$

$$6EI$$

$$4EI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

$$4EI$$

$$y = \left[ \frac{wL^2}{4EI} \right] x^2 - \left[ \frac{wL}{6EI} \right] x^3 + \left[ \frac{w}{24EI} \right] x^4$$

$$= \frac{wL^2 x^2}{4EI} - \frac{wL x^3}{6EI} + \frac{w x^4}{24EI}$$

$$= \frac{6wL^2 x^2 - 4wL x^3 + w x^4}{24EI}$$

$$GE = y = A + Bx + \frac{w}{24EI} [6L^2 x^2 - 4L x^3 + x^4]$$

$$\text{at } y=0, x=0, \frac{dy}{dx}=0$$

$$0 = A + B(0) + \frac{w}{24EI} [6L^2(0) - 4L(0) + 0]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI} [12L^2 x - 12L x^2 + 4x^3]$$

$$0 = B + \frac{w}{24EI} [12L(0) - 12L(0) + 4(0)]$$

$$B = 0$$

$$y_p = \frac{w}{24EI} [6L^2 x^2 - 4L x^3 + x^4]$$

$$y_p = \frac{w x^2}{24EI} [6L^2 - 4xL + x^2]$$

$$y_p = \frac{w x^2}{24EI} [x^2 - 4xL + 6L^2]$$

when  $x=L$

$$y_D = \frac{wL^2}{24EI} [L^2 - 4L^2 + 6L^2]$$

$$\frac{y}{\delta P} = \frac{wL^2}{24EI} [3L^2]$$

$$y = \frac{wL^4}{8EI}$$