Assignment 1
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COMPUTER ENGINEERING
ENG 281

1a) $\lim_{x \to \pi/2} \left[\frac{\left(x^2 - \frac{\pi}{4}\right)\left(\sin(\cos x)\right)}{x - \pi/2} \right.$

For the numerator using product rule
let $u = x^2 - \pi/4$ and $v = \sin(\cos x)$

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v\frac{du}{dx}$$

For $u = x^2 - \pi/4$

$$\frac{du}{dx} = 2x$$

For $v = \sin(\cos x)$
let $w = \cos x$ and $v = \sin w$

$$\frac{dw}{dx} = -\sin x \qquad \frac{dv}{dw} = \cos w$$

$$\frac{dv}{dx} = \frac{dw}{dx} \times \frac{dv}{dw} = -\sin x \cos w$$

$$\frac{dv}{dx} = -\sin(\cos(\cos x))$$

$$\frac{dy}{dx} = \left[x^2 - \frac{\pi}{4}\right] \cdot -\cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$$

$$\frac{dy}{dx} = \left[x^2 - \frac{\pi}{4}\right] \cdot -\cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$$

For the denominator, let $b = x - \pi/2$

$$\frac{db}{dx} = 1$$

$$= \frac{3\pi - 2}{6} \text{ or } \frac{\pi}{2} - \frac{1}{3}$$

$$\lim_{x \to \frac{\pi}{2}} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right] = \frac{3\pi-2}{6}$$

3)
$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{(2+\sqrt{3})-\cancel{2}}{2+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}}\right)\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

4)

$$= \cos(60) = 1/2$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = 1/2$$

4)
$$\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$$

$$= \lim_{x \to 4}\left[\frac{2x-8}{2x-5}\right]$$

$$= \lim_{x \to 4}\left[\frac{2(4)-8}{2(4)-5}\right] = \frac{8-8}{8-5}$$

$$= \frac{0}{3} = 0$$

$$\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = 0$$

series is convergent

a) $\frac{2}{2\times3}+\frac{2}{3\times4}+\frac{2}{4\times5}+\frac{2}{5\times6}+---$

Solution

$|U_n| =$

To test the series $\frac{2}{2\times3}+\frac{2}{3\times4}+\frac{2}{4\times5}+\frac{2}{5\times6}+---$

If we take our standard series

$\frac{2}{2^p}+\frac{2}{3^p}+\frac{2}{4^p}+\frac{2}{5^p}+---$

when $P=2$, we get

$\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\frac{2}{5^2}+---$

Since

$\frac{2}{2\times3}<\frac{2}{2^2};\ \frac{2}{3\times4}<\frac{2}{3^2};\ \frac{2}{4\times5}<\frac{2}{4^2}$

Therefore, the given series is convergent.

b) $\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+---$

Solution:

For Comparison test

Using standard series, when $P=2$

$\frac{2}{1^p}+\frac{2}{2^p}+\frac{2}{3^2}+\frac{2}{4^2}+---$

Since $P > 1$, the series converges

Therefore, the given series is convergent.

c) $U_n = \frac{1+2n^2}{1+n^2}$

Solution.

$U_n = \frac{1+2n^2}{1+n^2}$

So $\lim_{n \to \alpha} U_n = \lim_{n \to \alpha} \left[\frac{1/n^2 + \frac{2n^2}{n^2}}{1/n^2 + n^2/n^2}\right]$

$= \left[\frac{1/n^2 + \frac{2}{1}}{1/n^2 + \frac{1}{1}}\right]$

$= \frac{0+2}{0+1}$

$= 2$

$\lim_{n \to \alpha} U_n = 2$

Since $\lim_{n \to \alpha} U_n \neq 0$

Therefore, the series is divergent.

3) Find the range of Values of x for which the series below is absolutely convergent

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

Soln

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{|U_{n+1}|}{|U_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{\cancel{x^n} \cdot x}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

$$= \frac{x\left[\frac{8\cancel{n^3}}{\cancel{n^3}} + \frac{12\cancel{n^2}}{\cancel{n^3}} + \frac{6\cancel{n}}{n^2} + \frac{1}{n^3}\right]}{\frac{8\cancel{n^3}}{\cancel{n^3}} + \frac{36\cancel{n^2}}{\cancel{n^3}} + \frac{54\cancel{n}}{n^2} + \frac{27}{n^3}}$$

$$= \frac{x\left[8 + 12/n + 6/n^2 + 1/n^3\right]}{8 + 36/n + 54/n^2 + 27/n^3}$$

$$= \frac{x\,[8+0+0+0]}{8+0+0+0}$$

$$= \frac{\cancel{8}x}{\cancel{8}}$$

$$= x$$

~~For~~ $\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right| = x$

For absolute Convergence $\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right| < 1$

$\therefore$ Series Convergent when $-1 \leq x \leq 1$

4) Evaluate using L'Hopital's Rule

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Soln

$$= \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$= \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x + (-\sin x}{6} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6} \right]$$

$$= \frac{-1 + 0}{6}$$

$$= \frac{-1}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = \frac{-1}{6}$$

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$$\lim_{x \to \pi/2} \left(\frac{\pi^2}{2^2} - \frac{\pi}{4}\right)\left(-\cos\left(\cos\frac{\pi}{2}\right)\sin\frac{\pi}{2}\right) + \sin(\cos \pi/2) 2\pi/2$$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)(-1) + 0$$

$$= \left(\frac{\pi^2 - \pi}{4}\right) - 1 = \frac{-\pi^2 + \pi}{4}$$

$$= \frac{\pi(-\pi + 1)}{4}$$

$$\therefore \lim_{x \to \pi/2}\left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2}\right] = \frac{\pi(-\pi+1)}{4}$$

b) $$\lim_{x \to \frac{\pi}{2}} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x+1}\right)\right]$$

$$= \cancel{\ln}\left[\cancel{\exp}\left(\frac{3x^2 + 2x - 1}{x+1}\right)\right]$$

$$= \lim_{x \to \frac{\pi}{2}}\left[\frac{3\left(\frac{\pi}{2}\right)^2 + \cancel{2}\left(\frac{\pi}{\cancel{2}}\right) - 1}{\left(\frac{\pi}{2}\right) + 1}\right]$$

$$= \frac{\frac{3\pi^2}{4} + \frac{\pi}{1} - \frac{1}{1}}{\frac{\pi + 2}{2}} = \frac{3\pi^2 + 4\pi - 4}{\cancel{4}\, 2} \times \frac{\cancel{2}\,1}{\pi + 2}$$

$$= \frac{3\pi^2 + 4\pi - 4}{2(\pi + 2)} = \frac{\left(\frac{\pi}{1} - \frac{2}{3}\right)\cancel{(\pi + 2)}}{2\,\cancel{(\pi + 2)}}$$

$$= \frac{3\pi - 2}{3} \times \frac{1}{2}$$