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Dept:- Mechanical Engineering. Assignment I

Matric No:- 15/ENG06/014

Course:- ENG381 (Engineering Mathematics II).

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Note: It is a non-homogeneous equation.

$\therefore$ General solution = Complementary function + Particular integral

$\therefore$ Complementary function (C.F):

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

the auxillary equation becomes

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + 1m - 2 = 0$$

$$m(m-2) + 1(m-2) = 0$$

$$m+1 = 0 \quad \& \quad m-2 = 0$$

$$m = -1 \quad \& \quad m = 2 \quad \text{--- Real \& different root}$$

$\therefore$ C.F = $Ae^{-x} + Be^{2x}$

Particular integral (P.I):

$$y = C \quad \text{--- (i)}$$

$$dy/dx = 0 \quad \text{--- (ii)}$$

$$d^2y/dx^2 = 0 \quad \text{--- (iii)}$$

Substituting equ(i), (ii) & (iii) into the general equation.

$$d^2y/dx^2 - dy/dx - 2y = 8$$

$$0 - 0 - 2(c) = 8$$

$$-2c = 8$$

$$c = 8/-2$$

$$c = -4$$

$\therefore y = c$

$\therefore y = -4$

General solution = Complementary function + Particular Integral.

$$y = Ae^{-x} + Be^{2x} + (-4)$$

$$y = Ae^{-x} + Be^{2x} - 4$$

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2) $d^2y/dx^2 - 4y = 10e^{3x}$

Note:- It is a non-homogenous equation

$\therefore$ General solution = complementary solution + Particular integral.

$\therefore$ complementary function (C.F).

$$d^2y/dx^2 - 4y = 0$$

The auxillary equation becomes

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm\sqrt{4}$$

$$m = \pm 2$$

$$m_1 = 2 \quad m_2 = -2$$

$\therefore$ $C.F = Ae^{2x} + Be^{-2x}$

Particular integral

Assume P.I

$$y = Ce^{3x} \quad --(i)$$

$$dy/dx = 3Ce^{3x} \quad ---(ii)$$

$$d^2y/dx^2 = 9Ce^{3x} \quad --(iii)$$

Sub equ(i), (ii) & (iii) into the general equation:

$$\therefore \quad d^2y/dx^2 - 4y = 10e^{3x}$$

$$\therefore 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$e^{3x}(9C - 4C) = 10e^{3x}$$

$$9C - 4C = 10$$

$$5C = 10$$

$$C = 10/5$$

$$C = 2$$

$$\therefore y = Ce^{3x}$$

$$y = 2e^{3x}$$

$\therefore$ General solution = Complementary function + Particular integral.

$$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$$

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5) $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

Note: It is a non-homogenous equation.

$\therefore$ General Solution = Complementary Function + Particular Integral.

Complementary function (C.F).

$$d^2y/dx^2 + 2dy/dx + y = 0$$

The auxillary equation becomes.

$$m^2 + 2m + 1 = 0$$

$$\therefore m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$m = -1 \;\&\; m = -1 \quad \text{(twice)}.$$

$$\therefore C.F = e^{-x}(A + Bx)$$

Particular integral

Assumed P.I = $y = Ce^{-2x}$ --- (i)

$$dy/dx = -2Ce^{-2x} \text{ --- (ii)}$$

$$d^2y/dx^2 = 4Ce^{-2x} \text{ --- (iii)}$$

Sub equ(i), (ii) & (iii) into the general equation

$$d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$\therefore y = Ce^{-2x}$$

$$\therefore y = e^{-2x}$$

$\therefore$ General Solution = Complementary function + Particular Integral

$$y = e^{-x}(A + Bx) + e^{-2x}$$

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4) $d^2y/dx^2 + 25y = 5x^2 + x$

Note: It is a non-homogenous equation

$\therefore$ General Solution = Complementary Function + Particular integral

$\therefore$ Complementary function:

$$d^2y/dx^2 + 25y = 0$$

the auxcillary equation becomes:

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm\sqrt{-25}$$

$$m = \pm\sqrt{25} \cdot \sqrt{-1}$$

$$m = \pm 5j \text{ --- complex root.}$$

Comparing with $m = \alpha \pm \beta j$

$$\alpha = 0 \qquad \beta = 5.$$

$$\therefore C.F = e^{0x}(A\cos 5x + B\sin 5x)$$

$$\text{Since: } e^0 = 1$$

$$\therefore C.F = A\cos 5x + B\sin 5x$$

Particular integral:

Assumed P.I

$$y = Cx^2 + Dx + E \text{ --- (i)}$$

$$dy/dx = 2Cx + D. \text{ --- (ii)}$$

$$d^2y/dx^2 = 2C \text{ --- (iii)}$$

Sub equ(i), (ii) & (iii) into the general equation

$$d^2y/dx^2 + 25y = 5x^2 + x$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$25Cx^2 = 5x^2$$

$$25C = 5$$

$$C = 5/25$$

$$C = 1/5 \text{ --- (iv)}$$

$$25Dx = x$$

$$25D = 1$$

$$D = 1/25 \text{ --- (v)}$$

15/ENG06/014

Assignment II

Cont'd

4) $2C + 25E = 0$

Recall $C = 1/5$

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$25E = 2/5$

$E = 2/5 \times 1/25$

$E = \frac{2}{125}$ --- (vi)

∴ Sub equ(iv), (v) & (vi) into equ(i).

$y = Cx^2 + Dx + E$

$y = \frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}$

∴ General solution = Complementary function + Particular Integral

$y = A\cos 5x + B\sin 5x + \left(\frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}\right)$

∴ $y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}$

5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

Note: It is a non-homogenous equation

General solution = Complementary function + Particular Integral

∴ Complementary function.

$$d^2y/dx^2 - 2\,dy/dx + y = 0$$

The auxcillary equation becomes

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$m = 1 \text{ (twice)}.$$

$$\therefore C.F = e^{x}(A + Bx)$$

Particular integral.

Assumed P.I =

$$y = C\cos x + D\sin x \quad \text{---(i)}$$

$$dy/dx = -C\sin x + D\cos x \quad \text{---(ii)}$$

$$d^2y/dx^2 = -C\cos x - D\sin x \quad \text{---(iii)}$$

Sub equ(i), (ii) & (iii) into the general equation

$$d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$$

$$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$\therefore -C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$\Rightarrow 2C\sin x - 2D\cos x = 4\sin x.$$

Comparing co-efficient:

$$2C = 4$$

$$-2D = 0$$

$$C = 4/2 \quad \text{---(iv)}$$

$$D = 0 \quad \text{---(v)}$$

$$\therefore y = C\cos x + D\sin x$$

$$y = 2\cos x + 0\sin x$$

$$y = 2\cos x \quad \text{---(vi)}$$

∴ General solution = Complementary function + Particular integral

$$y = e^{x}(A + Bx) + 2\cos x$$

6) $d^2y/dx^2 + 4\, dy/dx + 5y = 2e^{-2x}$

Given that at $x=0$, $y=1$ & $dy/dx = -2$

Note; It is a non-homogenous equation

General Solution = Complementary Function + Particular Integral

:- Complementary Function:-

$$d^2y/dx^2 + 4\, dy/dx + 5y = 0$$

The auxiliary equation becomes:

$$m^2 + 4m + 5 = 0$$

Using quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a=1$, $b=4$ $c=5$

$$\frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$$

$$\Rightarrow \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$\sqrt{-4} = \sqrt{4} \times \sqrt{-1}$$

$$= 2j$$

$$\therefore \frac{-4 \pm 2j}{2}$$

$$= -2 \pm j$$

Comparing with $m = \alpha + \beta j$

$\alpha = -2$ $\beta = 1$

$$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

$$y = e^{-2x}(A\cos x + B\sin x)$$

:- Particular integral

Assumed P.I

$$y = Ce^{-2x}$$

But since Ce^{-2x} is both in the C.F & assumed P.I, we would multiply the P.I by x.

$\therefore y = xCe^{-2x}$

$y = Cxe^{-2x}$ --- (i)

$dy/dx = -2Cxe^{-2x}$ ----- (ii)

$d^2y/dx^2 = 4Cxe^{-2x}$ ------ (iii)

Sub equ (i), (ii) & (iii) into the general equation

$\therefore \quad d^2y/dx^2 + 4\, dy/dx + 5y = 2e^{-2x}$

$4Cxe^{-2x} + 4(-2Cxe^{-2x}) + 5(Cxe^{-2x}) = 2Ce^{-2x}$

$4Cxe^{-2x} - 8Cxe^{-2x} + 5Cxe^{-2x} = 2Ce^{-2x}$

$Cxe^{-2x}(4 - 8 + 5) = 2Ce^{-2x}$

$Cxe^{-2x}(1) = 2Ce^{-2x}$

$Cx = 2$

$C = 2/x$ ---- (iv)

Sub equ (iv) into (i)

$y = Cxe^{-2x}$

$y = \frac{2}{x} xe^{-2x}$

$\therefore y = 2e^{-2x}$

The general solution becomes:

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

To find the values of A & B

when $x = 0$, $y = 0$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1\,(A\cdot 1 + (B\cdot 0)) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

when $x = 0$ $dy/dx = -2$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y = e^{-2x}A\cos x + e^{-2x}B\sin x + 2e^{-2x}$

using product rule

$dy/dx = -e^{-2x}[A\sin x] - 2e^{-2x}[A\cos x] + e^{-2x}[B\cos x]$
$- 2e^{-2x}[B\sin x] - 4e^{-2x}$

$-2 = -e^{-2(0)}[A\sin(0)] - 2e^{-2(0)}[A\cos(0)] + e^{-2(0)}[B\cos(0)]$
$- 2e^{-2(0)}[B\sin(0)] - 4e^{-2(0)}$

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Assignment I

Cont'd:

6) $-2 = 0 - 2[A] + B - 0 - 4$

$-2 = -2A + B - 4$

Re-arranging:

$-2A + B = -2 + 4$

But recall

$A = -1$

$\therefore -2(-1) + B = 2$

$2 + B = 2$

$B = 2 - 2$

$B = 0$

Sub $A = -1$ & $B = 0$ into the equation.

$y = e^{-2x}(-1\cos x + (0)\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Note:, It is a non-homogenous equation.

General Solution = Complementary Function + Particular integral

Complementary function.

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$$

the auxcillary equation becomes.

$$3m^2 - 2m - 1 = 0$$

$m = 1$ & $m = -\frac{1}{3}$ --- Real & different root.

$C.F = Ae^{x} + Be^{-\frac{1}{3}x}$

Particular integral.

Assumed P.I =

$$y = Cx + D \text{ ---(i)}$$

$$\frac{dy}{dx} = C \text{ ---(ii)}$$

$$\frac{d^2y}{dx^2} = 0 \text{ ---(iii)}$$

$\therefore$ Sub equ(i), (ii) & (iii) into the general equation.

$\therefore$ $3(0) - 2(C) - (Cx + D) = 2x - 3$

$$0 - 2C - Cx - D = 2x - 3$$

comparing co-efficient.

$$-Cx = 2x$$

$$-C = 2$$

$$C = -2 \text{ ---(iv)}$$

$$-2C - D = -3$$

Recall $C = -2$.

$\therefore$ $-2(-2) - D = 3$

$$4 - D = 3$$

$$D = 4 - 3$$

$$D = 1 \text{ ---(v)}$$

$\therefore$ $y = -2x +$

$\therefore$ General Solution =>

$$y = Ae^{x} + Be^{-\frac{1}{3}x} - 2x + 1$$

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8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

It is a non-homogenous equation.

General Solution = Complementary function + Particular integral.

Complementary function.

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$$

The auxillary equation becomes.

$$m^2 - 6m + 8 = 0$$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$m = 4$ & $m = 2$ (real & different root).

$\therefore$ C.F = $Ae^{4x} + Be^{2x}$

Particular Integral

Assumed P.I

$$y = ce^{4x}$$

But since e^{4x} is both in the C.F & assumed P.I, we would multiply the P.I by x.

$$\therefore y = Cxe^{4x} \quad \text{---(i)}$$

using Product rule.

$$\frac{dy}{dx} = C\{x4e^{4x} + e^{4x}\}. \quad \text{---(ii)}$$

using Product rule.

$$\frac{d^2y}{dx^2} = C\{x16e^{4x} + 4e^{4x} + 4e^{4x}\}$$

$$= C\{x16e^{4x} + 8e^{4x}\} \quad \text{---(iii)}$$

Sub equ(i), (ii) & (iii) into the general equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$Cx16e^{4x} + C8e^{4x} - 6(Cx4e^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$$

$$Cx16e^{4x} + C8e^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$Cxe^{4x}(16 - 24 + 8) + Ce^{4x}(8-6) = 8e^{4x}$$

$$0 + Ce^{4x}(2) = 8e^{4x}$$

$$2C = 8$$

$$C = 8/2$$

$$C = 4$$

$\therefore y = Cxe^{4x}$

$y = 4xe^{4x}$

$\therefore$ General solution = Complementary function + Particular integral

$$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$