15/ENG09/002

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1. (i) $m^2 + 4m + 5 = 0$ (using auxiliary method)

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4 \pm j2}{2}$$

$$m = -2 \pm j$$

C.F: $y = e^{-2x}(A\cos\theta + B\sin\theta)$

P.I.: $y = C\cos\theta + D\sin\theta$

$$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$$

$$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$$

$$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5 = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5(\cancel{\theta\ 6\sin\theta}\ C\cos\theta + D\sin\theta) = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$$

$$\cos\theta(-C + 4D + 5C) + \sin\theta(-D - 4C + 5D) = 6\sin\theta$$

$$\cos\theta(4C + 4D) + \sin\theta(4D - 4C) = 6\sin\theta$$

Comparing both sides

$4C + 4D = 0$ ---- (1)

$-4C + 4D = 6$ --- (2)

$8D = 6$

$D = \frac{3}{4}$

Subst. into (1)

$4C + 3 = 0$

$C = -3/4$

$\therefore$ P.I: $y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

G.S: $y = e^{-2\theta}(A\cos\theta + B\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

(iii) At steady state $\theta = \infty$, $\frac{dy}{d\theta} = 0$

$$\frac{dy}{d\theta} = -2e^{-2\theta}(A\cos\theta + B\sin\theta) + e^{-2\theta}(-A\sin\theta + B\cos\theta) + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$\cancel{0 = -2e^{-2(\infty)}}$

$$0 = 0 + 0 + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$$-\frac{3}{4}\sin\theta = \frac{3}{4}\cos\theta$$

$$-\sin\theta = \cos\theta$$

Divide through by $\cos\theta$

$$-\frac{\sin\theta}{\cos\theta} = 1$$

$$-\tan\theta = 1$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

2. $$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

C.F:

~~Aug~~ Auxilliary equation.

$$m^2 = 0$$

$$m = \sqrt{0}$$

$$m = \pm 0 \text{ twice}$$

$$y = e^0(A + Bx)$$

$$y = A + Bx$$

Assume P.I.

$$y = Cx^2 + Dx^3 + Fx^4$$

$$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Fx^3$$

$$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Fx^2$$

$$EI(2C + 6Dx + 12Fx^2) = \frac{w}{2}(L-x)^2$$

$$2CEI - 6DEIx + 12FEIx^2 = \frac{w}{2}(L-x)^2$$

$$2CEI - 6DEIx + 12FEIx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$$

$$4CEI + 12DEIx + 24FEIx^2 = W(L^2 - 2Lx + x^2)$$

$$4CEI + 12DEIx + 24FEIx^2 = WL^2 - 2WLx + Wx^2$$

Comparing coefficient.

x^2: $$24FEI = W$$

$$F = \frac{W}{24EI}$$

x: $$12DEI = -2WL$$

$$D = -\frac{WL}{6EI}$$

constant: $$4CEI = WL^2$$

$$C = \frac{WL^2}{4EI}$$

$$\therefore y = \frac{WL^2}{4EI}x^2 - \frac{WL}{6EI}x^3 + \frac{W}{24EI}x^4$$

$$\therefore G.S = y = A + Bx + \frac{WL^2}{4EI}x^2 - \frac{WL}{6EI}x^3 + \frac{W}{24EI}x^4$$

$$\frac{dy}{dx} = B + \frac{WL^2}{2EI}x - \frac{WL}{2EI}x^2 + \frac{W}{6EI}x^3$$

at $\frac{dy}{dx} = 0$, $x = 0$

$$0 = B$$

at $y = 0$, $x = 0$

$$A = 0$$

G.S:

$$y = \frac{WL^2}{4EI}x^2 - \frac{WL}{6EI}x^3 + \frac{W}{24EI}x^4$$

at $x = L$

$$y = \frac{WL^4}{4EI} - \frac{WL^4}{6EI} + \frac{WL^4}{24EI}$$

$$y = \frac{6WL^4 - 4WL^4 + WL^4}{24EI}$$

$$y = \frac{3WL^4}{24EI}$$

$$y = \frac{WL^4}{8EI}$$

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