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Assignment 2

1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

i) Cf : $y = m^2 + 4m + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm 2j}{2} = -2 \pm j$

$\alpha = -2, \beta = 1$

$y = e^{\alpha\theta}[A\cos\beta\theta + B\sin\beta\theta]$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

for $6\sin\theta$

assumed PI ÷ $C\cos x + D\sin x$

$y = C\cos\theta + D\sin\theta$ ... (1)

$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$ ... (2)

$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$ ... (3)

substituting eqn (1), (2) and (3) into the original equation

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$4C\cos\theta + 4D\sin\theta - 4C\sin\theta + 4D\cos\theta = 6\sin\theta$

Equating coefficient ;

$4C + 4D = 0$; $C = -D$

$4D - 4C = 6$

$4D - 4(-D) = 6$

$4D + 4D = 6$

$8D = 6$

$D = 3/4$

$C = -D \; ; \; C = -3/4$

$\therefore y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

General Solution:

$$y = CF + PI$$

$$y = e^{-2\theta}[A\cos\theta + B\sin\theta] - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$$

ii) neglecting the complementary function:

~~$y = e^{-2x}$~~ $y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

iii) at $\theta = \infty$ and $dy/d\theta = 0$; $dy/d\theta = e^{-2\theta}(-A\sin\theta + B\cos\theta) + [A\cos\theta + B\sin\theta]-2e^{-2\theta} + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$, $-\frac{3}{4}\cos\theta = \frac{3}{4}\sin\theta$, $-\cos\theta = \sin\theta$ divide both sides by $-\cos\theta$

$-\tan\theta = 1$, $\tan\theta = -1$, $\theta = \tan^{-1}(-1)$

$\theta = -45^0$

2) $EI\frac{d^2y}{dx^2} = \frac{w(L-x)^2}{2}$

$EIm^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$y = e^{0x}(A+Bx)$

$y = A + Bx$ ... CF

To obtain periodic integral

$y = Cx^2 + Dx^3 + Ex^4$ ... (1)

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Ex^3$ ... (2)

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Ex^2$ ... (3)

Substituting eqn (3) into the original equation

$EI(2C + 6Dx + 12Ex^2) = \frac{w}{2}(L-x)^2$

$2CEI + 6DEIx + 12EIEx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4CEI + 12DEIx + 24EIEx^2 = w(L^2 - 2Lx + x^2)$

Equating coefficient

$24EIE = w$

$E = \frac{w}{24EI}$ ... (4)

$12DEI = -2wL$

$D = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$ ... (5)

$4CEI = wL^2$

$C = \frac{wL^2}{4EI}$ ... (6)

Substituting eqn 4, 5 and 6 into eqn (1)

$y = \left(\frac{wL^2}{4EI}\right)x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$

$$= \frac{wl^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$= \frac{6wl^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$= \frac{w}{24EI}[6l^2x^2 - 4Lx^3 + x^4] \quad \text{.... PI}$$

General Solution

$$y = A + Bx + \frac{w}{24EI}(6l^2x^2 - 4lx^3 + x^4)$$

at $y = 0$, $x = 0$, $\frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}[6L^2(0) - 4L(0) + 0]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}(12L^2x - 12lx^2 + 4x^3)$$

$dy/dx = 0$, $x = 0$

$$0 = B + \frac{w}{24EI}[12(0) - 12(0) + 4(0)]$$

$$B = 0$$

$$\therefore y = \frac{w}{24EI}(6l^2x^2 - 4lx^3 + x^4)$$

$$y = \frac{wx^2}{24EI}[6L^2 - 4lx + x^2]$$

$$y = \frac{wx^2}{24EI}[x^2 - 4lx + 6l^2]$$

when $x = L$

$$y = \frac{wl^2}{24EI}(l^2 - 4l^2 + 6l^2) = \frac{wl^2}{24EI}(3l^2)$$

$$y = \frac{wl^4}{8EI}$$