Bello Miriam Ayuba
15/sci04/010
Chemical Engineering
Eng 381 - Engineering Maths III

1. $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = 1 \quad b = 4 \quad c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$\alpha = -2 \qquad \beta = 1$

CF : $y = e^{\alpha\theta}(A\cos\beta\theta + B\sin\beta\theta)$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

PI : $y = C\cos\theta + D\sin\theta$

$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C\cos\theta + 4D\cos\theta + 5C\cos\theta - D\sin\theta - 4C\sin\theta + 5D\sin\theta = 6\sin\theta$

$-C\cos\theta + 4D\cos\theta + 5C\cos\theta = 0$

$4A\cos\theta + 4C\cos\theta = 0$

$4A + 4C = 0$

$4C = -4A$

$C = -A$

$-A\sin\theta - 4C\sin\theta + 3A\sin\theta = 6\sin\theta$

$-4C\sin\theta + 4A\sin\theta = 6\sin\theta$.

$-4C + 4A = 6$

Substitute $C = -A$ into the equation above

$-4(-A) + 4A = 6$

$4A + 4A = 6$

$8A = 6$

$A = 6/8$

$A = 3/4$.

Recall $C = -A$

$C = -3/4$

PI : $y = -3/4\cos\theta + 3/4\sin\theta$.

GS = CF + PI

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$.

ii $y = -3/4\cos\theta + 3/4\sin\theta$.

$y = -0.75\cos\theta + 0.75\sin\theta$.

iii at steady state, $\frac{dy}{d\theta} = 0$ and $\theta = \infty$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$.

$\frac{dy}{d\theta} = u\,dv + v\,du \left(+3/4\cos\theta + 3/4\sin\theta\right)$

$u = e^{-2\theta}$.

$du = -2e^{-2\theta}$.

$v = A\cos\theta + B\sin\theta$.

$dv = -A\sin\theta + B\cos\theta$.

$\frac{dy}{d\theta} = -2e^{-2\theta}(A\cos\theta + B\sin\theta) + e^{-2\theta}(-A\sin\theta + B\cos\theta)$

$+ 3/4\sin\theta + 3/4\cos\theta$.

At steady state, $\theta = \infty$ and $dy/d\theta = 0$

$e^{-2\theta} = e^{-2\infty} = 0$

$\frac{dy_{ss}}{d\theta} = 0$

$0 = 3/4 \sin\theta + 3/4 \cos\theta -$

$\frac{3/4 \sin\theta}{3/4 \cos\theta} = \frac{-3/4 \cos\theta}{3/4 \cos\theta}$

$\frac{\sin\theta}{\cos\theta} = -1$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45^{\circ}$ //

2. $EI \frac{d^2y}{dx^2} = \frac{w}{2}((-x)^2$ (1)

$EI\ m^2 = 0$

$m^2 = 0$

$m = 0$

$y = e^{0x}(A + Bx)$ (CF)

P.I : $y = Cx^2 + Dx^3 + Ex^4$

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Ex^3$

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Ex^2$

$EI(2C + 6Dx + 12Ex^2) = w/2((-x)^2$

$EI(2C + 6Dx + 12Ex^2) = \frac{w}{2}(l^2 - 2lx + x^2)$

$EI\,2C + EI\,6Dx + EI\,12Ex^2 = \frac{wl^2}{2} - lwx + \frac{wx^2}{2}$

$EI\,2C = \frac{wl^2}{2}$

$C = \frac{wl^2}{2} \times \frac{1}{2EI}$

$C = \frac{wl^2}{4EI}$

$EI\,6D = -lw$

$D = \frac{-lw}{6EI}$

$EI\,12E = \frac{w}{2}$

$$E = \frac{w}{2} \times \frac{1}{12EI}$$

$$E = \frac{w}{24EI}$$

$$PI: y = \left[\left(\frac{wl^2}{4EI}\right)x^2 + \left(\frac{-wl}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)\right]x^4$$

$$y = \frac{wl^2x^2 \cdot 6 - 4wlx^3 + wx^4}{24EI}$$

when $y = 0$ & $x = 0$

$$0 = A + B(0) + \frac{w}{24EI}(0)$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}(12l^2x - 12lx^2 + 4x^3)$$

when $\frac{dy}{dx} = 0$

$$0 = B + \frac{w}{24EI}(0)$$

$$B = 0$$

$$GS: y = 0 + 0 + \frac{w}{24EI}(6l^2x^2 - 4lx^3 + x^4)$$

$$y = \frac{w}{24EI}(6l^2x^2 - 4lx^3 + x^4)$$

when $x = l$

$$y = \frac{w}{24EI}(6l^2(l)^2 - 4l(l)^3 + (l)^4)$$

$$y = \frac{w}{24EI}(6l^4 - 4l^4 + l^4)$$

$$y = \frac{w}{24EI}3l^4$$

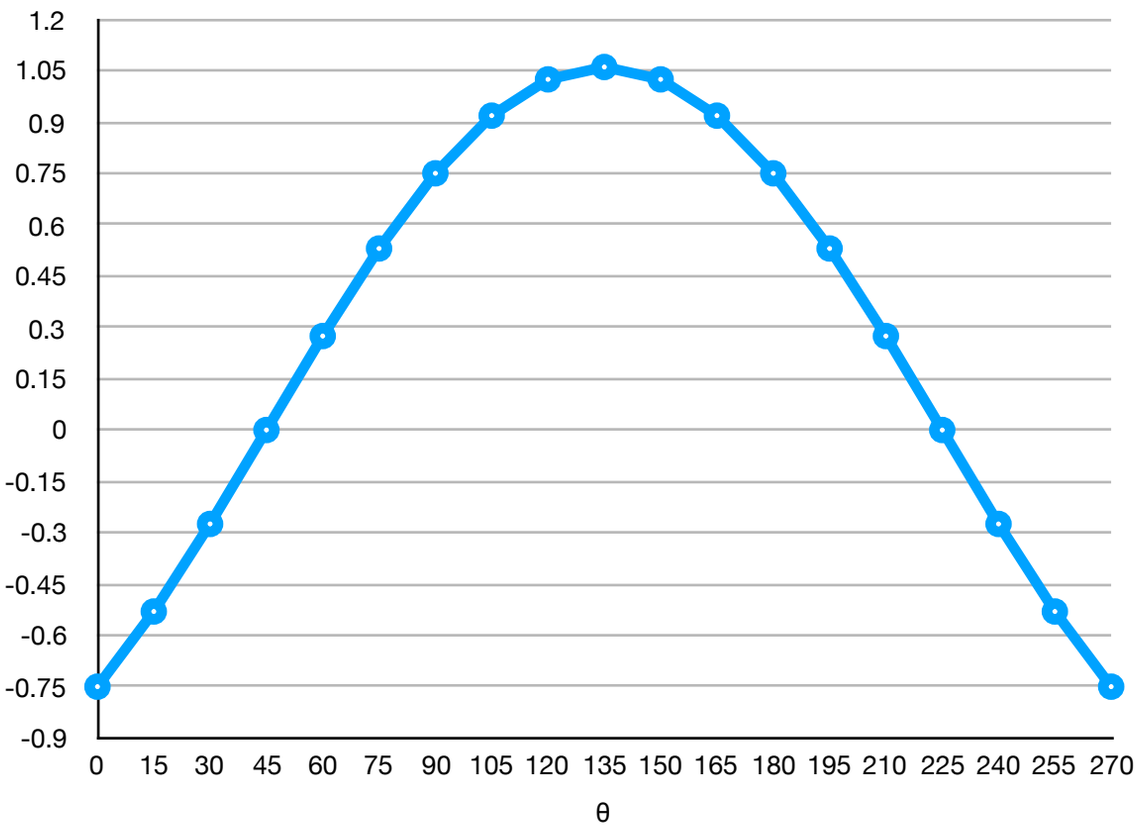
$$y = \frac{3wl^4}{8 \cdot 24EI}$$

$$y = \frac{wl^4}{8EI}$$

TABLE OF Y AGAINST θ

θ	Y
0	-0.75
15	-0.530330085889911
30	-0.274519052838329
45	-0.000000000000000
60	0.274519052838329
75	0.530330085889911
90	0.75
105	0.918558653543692
120	1.02451905283833
135	1.06066017177982
150	1.02451905283833
165	0.918558653543692
180	0.75
195	0.530330085889911
210	0.274519052838329
225	0.000000000000000
240	-0.274519052838328
255	-0.530330085889911
270	-0.75

GRAPH OF Y AGAINST θ



GRAPH OF Y AGAINST θ