7) $3\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - y = 2x - 3$

$3d^2y - 2dy/dx - y = 0$

Complementary function:

$3m^2 - 2m - 1 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-2) \pm \sqrt{(2)^2 - 4(3 \times -1)}}{2(3)}$

$= \dfrac{2 \pm \sqrt{4 + 12}}{6} = \dfrac{2 \pm \sqrt{16}}{6} = \dfrac{1}{3} \pm \dfrac{4}{6} = \dfrac{1}{3} \pm \dfrac{2}{3}$

$m_1 = \dfrac{1+2}{3} = 1 \qquad m_2 = \dfrac{1-2}{3} = \dfrac{-1}{3}$

$y = Ae^x + Be^{-x/3}$

partial integration:

$y = Cx + \Delta$

$\dfrac{dy}{dx} = C$

$\dfrac{d^2y}{dx^2} = 0$

$\therefore 3(0) - 2(c) - Cx - D = 2x - 3$

$-2c - Cx - \Delta = 2x - 3$

Comparing LHS & RHS

$-C \leq 2 \qquad\qquad -2c - \Delta = -3$

$C = 2 \qquad\qquad -2(2) - \Delta = -3$

$\qquad\qquad\qquad -4 - \Delta = -3$

$\qquad\qquad\qquad \Delta = 4 + 3 = 7$

$\Rightarrow y = -2x + 7$

General Solution:

$y = Ae^x + Be^{-\frac{x}{3}} - 2x + 7$

$3)\ \dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 2e^{-2x}$

Complementary function

$\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-4 \pm \sqrt{16 - 20}}{2} = \dfrac{-4 \pm j\sqrt{4}}{2} = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

Partial Integration

$y = Ce^{-2x}$

$\dfrac{dy}{dx} = -2Ce^{-2x}$

$\dfrac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2(Ce^{-2x})) + 5(Cce^{-2x})$

$4Ce^{-2x} - 8Ce^{-2x} \quad (5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$\therefore y = 2e^{-2x}$

General solution;

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x = 0, y = 1$

$\Rightarrow 1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$A = 1 - 2 = -1$

4) $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\dfrac{d^2y}{dx^2} + 25y = 0$

Complementary function:

$m^2 + 25 = 0$

$m^2 = 25$

$m = \pm 5$

$y = A\cos 5x + B\sin 5x$

Particular integration:

$y = Cx^2 + Dx + E$

$\dfrac{dy}{dx} = 2Cx + D$

$\dfrac{d^2y}{dx^2} = 2C$

$2C + 25C(x^2 + Dx + E) = 5x^2 + x$

$2C + 25(x^2 + 25Dx + 25E = -5x^2 + x$

Complementary LHS & RHS:

$25x = 5$ $25A = 1$ $2C + 25E = 0$

$C = \dfrac{1}{5}$ $A = \dfrac{1}{25}$ $2\left(\dfrac{1}{5}\right) + 25E = 2$

                                    $E = \dfrac{-2}{5} \div 25 = \dfrac{-2}{125}$

$y = \dfrac{x^2}{5} + \dfrac{x}{25} - \dfrac{2}{125}$

General solution:

$y = A\cos 5x + B\sin 5x + \dfrac{x^2}{5} + \dfrac{x}{25} - \dfrac{2}{125}$

Ass1
Charles C. Ndudi-Wali
15/ENQ02/037

1) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 8$

$\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4a}}{2a}$

$= \dfrac{-1 \pm \sqrt{1 - 4(1 \times -2)}}{2(1)}$

$= \dfrac{1 \pm \sqrt{1 - 4(1 \times -2)}}{2}$

$= \dfrac{-1 \pm \sqrt{1 + 8}}{2}$ $\qquad = \dfrac{-1 \pm 3}{2}$

$m_1 = \dfrac{-1 + 3}{2} = 1 \qquad m_2 = \dfrac{-1 - 3}{2} = -2$

$y = Ae^z + A^{-2x}$ $\leftarrow$ C.7

P.I

$y = C$

$\dfrac{dy}{dx} = 0$

$\dfrac{d^2y}{dx^2} = 0$

$\therefore -2c = 8$

$C = -4$

$\Rightarrow y = -4$

Hence the general solution = C.7 + P.I $= y = Ae^z + Be^{-x} - 4$

2) $\dfrac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\dfrac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4$

$m = \pm 2$

$y = A\cosh 2x + B\sinh 2x \quad \Leftarrow$ Complementing funct

Partial integration

$y = Ce^{3x}$

$\dfrac{dy}{dx} = 3Ce^{3x}$

$\dfrac{d^2y}{dx^2} = 9Ce^{3x}$

$\therefore \quad 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$\qquad 5Ce^{3x} = 10e^{3x}$

$\qquad C = 2$

$\qquad \therefore y = 2e^{3x}$

General Solution,

$\qquad y = A\cosh 2x + B\sinh 2x + 2e^{3x} \,/\!/$

1) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 8y = 8e^{4x}$

$\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 8y = 0$.

Complementing function.

$m^2 - 6m + 8 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(6) \pm \sqrt{(-6)^2 - 4(1 \cdot 8)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{36 - 52}}{2} = \dfrac{6 \pm \sqrt{4}}{2} = 3 \pm 1$

$M_1 = 3 + 1 = 4 \qquad M_2 = 3 - 1 = 2$.

$\Rightarrow y = Ae^{2x} + Be^{4x}$

Practical Integration.

$y = Ce^{4x}$

$\dfrac{dy}{dx} = 4Ce^{4x}$

$\dfrac{d^2y}{dx^2} = 16Ce^{4x}$

$\Rightarrow 16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$0C = 8e^{4x}$

$C = 0$

$y = 0$.

General solution;

$y = Ae^{2x} + Be^{4x}$ //

3) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = e^{-2x}$

Complementary function

$\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$(m+1)(m+1)$

$m = -1$

$y = e^{-x}(A + Bx)$

partial integration:

$y = Ce^{-2x}$

$\dfrac{dy}{dx} = -2Ce^{-2x}$

$\dfrac{d^2y}{dx^2} = +4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore y = e^{-2x}$

General solution

$y = e^{-x}(A + Bx) + e^{-2x}$ //

$$\frac{dy}{dx} = \frac{d(uv)}{dx}$$

$u = e^{-2x}$ $\qquad V = A\cos x + B\sin x$

$\frac{du}{dx} = -2e^{-2x}$ $\qquad \frac{dv}{dx} = -A\sin x + B\cos x$

$e^{-2x}(-A\sin x + B\cos x) + (A\cos x + A\sin x)(-2e^{-2x})$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x) \cdot (-2e^{-2x})$

when $\frac{dy}{dx} = -2, \; x = 0, \; y = 1$

$\Rightarrow -2 = e^{-2(0)}(-A\sin(0) + B\cos(0)) + (A\cos 0 + B\sin 0)(-2e^{0})$

$-2 = 1(B + A(-2) - 4$

$-2 = B - 2A - 4$

$2 = B - 2A$

$B = 2 + 2A \qquad \text{but } A = -1$

$B = 2 + 2(-1)$

$B = 0$

$y = e^{-2x}(-\cos x + 0) = e^{-2x}(\cos x)$

$$5)\ \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

Complementary function:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2$$

$$m = 1$$

$$y = e^x(A + Bx)$$

partial Integration:

$$y = C\cos x + D\sin x$$

$$\frac{dy}{dx} = -C\sin x + D\sin x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$$\Rightarrow -C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$$

$$\cos x(-2D) + \sin x(2C) = 4\sin x$$

Comparing LHS ⇒ RHS

$$2C = 4$$

$$-2D = 0$$

$$C = 2$$

$$D = 0$$

∴ $y = 2\cos x$

General solution,

$$y = e^x(A + Bx) + 2\cos x$$