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1. The parametric equations of a curve as given in eqn (1) and (2)

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

In terms of t , determine

- i. An expression for the radius of curvature (R), and
- ii. Expressions for the coordinates (h, k) of the centre of curvature.

$$1. \quad R = \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{d^2y/dx^2}$$

$$\frac{dy}{dx} = ? \quad \frac{d^2y}{dx^2} = ?$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$y = \sin t - t \cos t$$

$$\frac{dy}{dt} = \cos t - [-t \sin t + \cos t]$$

$$= \cos t + t \sin t - \cos t$$

$$\frac{dy}{dt} = t \sin t$$

$$x = \cos t + t \sin t$$

$$\frac{dx}{dt} = -\sin t + [t \cos t + \sin t]$$

$$= -\sin t + t \cos t + \sin t$$

$$\frac{dx}{dt} = t \cos t$$

$$\frac{dy}{dx} = \frac{t \sin t}{t \cos t} = \frac{\sin t}{\cos t}$$

$$\frac{dy}{dx} = \frac{t \sin t}{t \cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{dt}{dx}$$

$$= \frac{d}{dt} \left(\frac{\sin t}{\cos t} \right) \times \frac{dt}{dx}$$

$$= \frac{\cos t (\cos t) - \sin t (-\sin t)}{(\cos t)^2} \times \frac{1}{t \cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t \cos t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t \cos^2 t}$$

$$\begin{aligned}
 R &= \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \\
 &= \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \div \int \frac{dy}{dx} \\
 &= \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \div \frac{1}{t \cos^3 t} \\
 &= \left[1 + \left(\frac{\sin t}{\cos t} \right)^2 \right]^{3/2} \div \frac{1}{t \cos^3 t} \\
 &= \left[1 + \frac{\sin^2 t}{\cos^2 t} \right]^{3/2} \times t \cos^3 t \\
 &= \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \right]^{3/2} \times t \cos^3 t \\
 &= \left(\frac{1}{\cos^2 t} \right)^{3/2} \times t \cos^3 t \\
 &= \frac{1}{(\cos^2 t)^{3/2}} \times t \cos^3 t \\
 &= \frac{t \cos^3 t}{\cos^3 t} \\
 &= t
 \end{aligned}$$

ii) Centre of curvature

$$h = x - R \sin \theta \quad \text{--- (1)}$$

$$k = y + R \cos \theta \quad \text{--- (2)}$$

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

$$R = t, \theta = t$$

Sub x, y, R, θ into eqn (1) and eqn (2)

$$h = (\cos t + t \sin t) - t \sin t$$

$$h = \cos t$$

$$k = (\sin t - t \cos t) + t \cos t$$

$$k = \sin t$$

$$(h, k) = [\cos t, \sin t]$$