Matric no: 15/[illegible] Assignment 1

Department: chemical Engineering

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Sol

Non homogeneity

$y'' - y' - 2y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

Substitute the above into

$y'' - y' - 2y = 0$

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$

$e^{kx}(k^2 - k - 2) = 0$

$k^2 - k - 2 = 0$

$k^2 - 2k + k - 2 = 0$

$(k^2 - 2k) + (k - 2) = 0$

$k(k-2) + 1(k-2) = 0$

$(k+1)(k-2) = 0$

$k + 1 = 0$

$k - 2 = 0$

$k_1 = -1$ and $k = 2$

$y_1 = e^{k_1 x} = e^{-x}$

$y_2 = e^{k_2 x} = e^{2x}$

$y = c_1y_1 + c_2y_2$

$y = c_1 x e^{-x} + c_2 x e^{2x}$

$y = c_1e^{-x} + c_2e^{2x}$

$y' = 0 \quad y'' = 0$

$y_p = c$

Substitute into $y'' - y' - 2y = 8$

$0 - 0 - 2c = 8$

$\frac{-2c}{-2} = \frac{8}{-2}$

$c = -4$

$\therefore y = 4e^{-x} + c_2e^{2x} + c$

$y = c_1e^{-x} + c_2e^{2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Sol

$y'' - 4y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} - 4e^{kx} = 0$

$e^{kx}(k^2 - 4) = 0$

$k^2 - 4 = 0$

$k^2 = 4$

$= \sqrt{4}$

$k_1 = +2$ and $k_2 = -2$

$y_1 = e^{k_1 x} = e^{-2x}$

$y = c_1y_1 + c_2y_2$

$y = c_1 x e^{2x} + c_2 x e^{-2x}$

$y = c_1e^{2x} + c_2e^{-2x}$

$y_p = Ae^{3x}$

$y' = 3Ae^{3x}$

$y'' = 9Ae^{3x}$

Substitute into $y'' - 4y = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$e^{3x}(9A - 4A) = 10e^{3x}$

$\frac{5A}{5} = \frac{10}{5}$

$A = 2$

$\therefore y = c_1e^{2x} + c_2e^{-2x} + Ae^{3x}$

$y = c_1e^{2x} + c_2e^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Solution

$y'' + 2y' + y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

Substitute into $y''+2y'+y=0$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$e^{kx}(k^2+2k+1) = 0$

$k^2+2k+1 = 0$

$(k^2+k)+(k+1) = 0$

$k(k+1)+1(k+1) = 0$

$(k+1)(k+1) = 0$

$k+1 = 0$

$k+1 = 0$

$k_1 = -1$

$k_2 = -1$

$y_1 = e^{k_1x} = e^{-x}$

$y_2 = e^{k_2x} = e^{-x}$

$y = c_1y_1 + c_2y_2$

$y = c_1e^{-x} + c_2e^{-x}$

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$.

Substitute into $y''+2y'+y=e^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$\cancel{4Ae^{-2x}} - \cancel{4Ae^{-2x}} + A\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$A = 1$.

$y = c_1e^{-x} + c_2e^{-x} + e^{-2x}$

$y = e^{-x}(c_1+c_2) + e^{-2x}$.

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Solution

$y'' + 25y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} + 25e^{kx} = 0$

$e^{kx}(k^2+25) = 0$

$k^2+25 = 0$

$k^2 = -25$

$\sqrt{k^2} = \pm\sqrt{-25}$

$k_1 = +5i$

$k_2 = -5i$

$y_1 = e^{k_1x} = e^{5ix}$

$y_2 = e^{k_2x} = e^{-5ix}$

$y = c_1y_1 + c_2y_2$

$y = c_1e^{5xi} + c_2e^{-5xi}$

$y_p = Ax^2 + Bx + C$

$y' = 2Ax + B$

$y'' = 2A$

Substitute into $y'' + 25y = 5x^2 + x$

$2A + 25(Ax^2+Bx+C) = 5x^2+x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25A\cancel{x^2} = 5\cancel{x^2}$

$A = \frac{5}{25} = \frac{1}{5}$ — $*$

$25B\cancel{x} = \cancel{x}$

$B = \frac{1}{25}$ — $*$

$2A + 25C = 0$. where $A = \frac{1}{5}$

$2(\frac{1}{5}) + 25C = 0$

$25C = -\frac{2}{5}$ multiply both side by $\frac{1}{25}$

$C = -\frac{2}{125}$

$y_p = Ax^2 + Bx + C = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

(5

$y = C_1e^{5xi} + C_2e^{-5xi} + \frac{1}{125}(25x^2 + 5x - 2)$.

$y = C_1\cos 5t + C_2\sin 5t + \frac{1}{125}(25x^2 + 5x - 2)$.

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Solution:

$y'' - 2y' + y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} - 2ke^{kx} + e^{kx} = 0$

$e^{kx}(k^2 - 2k + 1) = 0$

$k^2 - 2k + 1 = 0$

$(k^2 - k - k + 1) = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k - 1 = 0$

$k - 1 = 0$

$k_1 = 1, \quad k_2 = 1.$

$y_1 = e^{k_1x} = e^x$

$y_2 = e^{k_2x} = e^x$

$y = C_1y_1 + C_2y_2 = C_1e^x + C_2e^x$

$y_p = A\sin x + B\cos x.$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x.$

Substitute into $y'' - 2y' + y = 0$

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos B + 2B\sin x + A\sin x + B\cos x = 4\sin x$

$(-A + 2B + A)\sin x + (-B - 2A + B)\cos x = 4\sin x$

$2B\sin x = 4\sin x$

$B = \frac{4}{2} = \underline{\underline{2}}$

$-2A\cos x = 0\cos x$

$A = 0$

$y = C_1e^x + C_2e^x + y_p$

$y = C_1e^x + C_2e^x + A\sin x + B\cos x$

$y = C_1e^x + C_2e^x + 0\sin x + 2\cos x$

$y = C_1e^x + C_2e^x + 2\cos x$

$y = e^x(C_1 + C_2) + 2\cos x.$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ $\quad x = 0, y = 1, \frac{dy}{dx} = -2.$

Solution

$y'' + 4y' + 5y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}.$

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$e^{kx}(k^2 + 4k + 5) = 0$

$k^2 + 4k + 5 = 0$

using $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

where $a = 1, b = 4, c = 5$

$k = \frac{-4 \pm \sqrt{(4)^2 - 4\times 1\times 5}}{2\times 1}$

$k = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$k = \frac{-4 \pm \sqrt{-4}}{2}$

$k = -4 \pm 2i$

$k_1 = \frac{-4 + 2i}{2} = \frac{2(-2 + i)}{2}$

$k_1 = -2 + i$

$k_2 = -2 - i$

$y_1 = e^{k_1x} = e^{(-2+i)x}$

$y_2 = e^{k_2x} = e^{(-2-i)x}.$

$Y_1 = e^{-2x} \cdot e^{ix} + \cancel{e^{-2x}(A\cos x + B\sin x)}$

$Y_2 = e^{-2x} \cdot e^{-ix}$

$Y = C_1Y_1 + C_2Y_2$

$Y = C_1(e^{-2x}e^{ix}) + C_2(e^{-2x} e^{-ix})$

$Y = e^{-2x}[C_1e^{ix} + C_2 e^{-ix}]$

$Y = e^{-2x}[C_1\cos x + C_2\sin x]$

$Y_P = Ae^{-2x}$

$Y' = -2Ae^{-2x}$

$Y'' = 4Ae^{-2x}$

Substitute into $Y''+4Y'+5Y = 2e^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$e^{-2x}(4A-8A+5A) = 2e^{-2x}$

$A = 2$

$\therefore Y = e^{-2x}[C_1\cos x + C_2\sin x] + 2e^{-2x}$

at $x=0$ $Y=1$

$1 = e^{-2(0)}[C_1\cos(0) + C_2\sin(0)] + 2e^{-2(0)}$

$1 = 1[C_1+0]+2$

$1 = C_1+2$

$1-2 = C_1$

$C_1 = -1$

to get $C_2$.

$Y' = -2e^{-2x}[-C_1\sin x + C_2\cos x] - 4e^{-2x}$

at $x=0$ $Y=-2$.

$-2 = -2e^{-2(0)}[-C_1\sin(0)+C_2\cos(0)] - 4e^{-2(0)}$

$-2 = -2[0+C_2]-4$

$-2 = -2 \times C_2 - 4$

$-2+4 = -2C_2$

$2 = -2C_2$

$C_2 = -1$

$Y = e^{-2x}[C_1\cos x + C_2 \sin x] + 2e^{-2x}$

Substitute $C_1=-1$ & $C_2=-1$.

$Y = e^{-2x}[\cos x - \sin x] + 2e^{-2x}$

$Y = e^{-2x}[-\cos x - \sin x + 2]$

$Y = e^{-2x}[2-\cos x - \sin x]$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x-3$

Solution

$3y''-2y'-y=0$

$y=e^{kx}$

$y' = ke^{kx}$

$y''=k^2e^{kx}$

$3k^2e^{kx} - 2ke^{kx} - e^{kx} = 0$

$e^{kx}(3k^2-2k-1)=0$

$3k^2-2k-1=0$

$(3k^2-3k)+(k-1)=0$

$3k(k-1)+1(k-1)=0$

$(3k+1)(k-1)=0$

$3k+1=0$

$k_1=-\frac{1}{3}$

$k-1=0$

$k_2=1$

$Y_1 = e^{k_1x} = e^{-\frac{1}{3} x}$

$Y_2 = e^{k_2x}=e^x$

$Y = C_1Y_1+C_2Y_2$

$Y = C_1e^{-\frac{1}{3} x} + C_2e^x$

$y_p = Ax - Bx^2$
$y' = A - 0 + 0$
$y'' = 0$
Substitute into $3y'' - 2y' - y = 2x - 3$
$0 - 2A - Ax - B = 2x - 3$
$-Ax = 2x$
$A = -2$ ---*
$-2A - B = -3$ where $A = -2$
$-2(-2) - B = -3$
$4 - B = -3$
$4 + 3 = B$
$B = 7$ ---*

$y_p = Ax - B = -2x + 7$

$\therefore y = C_1e^{-\frac{1}{3}x} + C_2e^{x} + y_p$

$\therefore y = C_1e^{-\frac{1}{3}x} + C_2e^{x} - 2x + 7$

3) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Solution

$y'' - 6y' + 8y = 0$
$y = e^{kx}$
$y' = ke^{kx}$
$y'' = k^2e^{kx}$
$k^2e^{kx} - 6ke^{kx} + 8e^{kx} = 0$
$e^{kx}(k^2 - 6k + 8) = 0$
$k^2 - 6k + 8 = 0$
$(k^2 - 2k - 4k + 8) = 0$
$k(k-2) - 4(k-2) = 0$
$(k-4)(k-2) = 0$
$k - 4 = 0$
$k - 2 = 0$

$k_1 = 4$
$k_2 = 2$
$y_1 = e^{k_1x} = e^{4x}$
$y_2 = e^{k_2x} = e^{2x}$
$y = C_1y_1 + C_2y_2$
$y = C_1e^{4x} + C_2e^{2x}$

$y_p = Ae^{4x}$
$y' = 4Ae^{4x}$
$y'' = 16Ae^{4x}$
Substitute into $y'' - 6y' + 8y = 0$
$16Ae^{4x} - 6(4Ae^{4x}) + 8Ae^{4x} = 8e^{4x}$
$16Ae^{4x} - \cancel{24Ae^{4x}} + \cancel{8Ae^{4x}} = 8e^{4x}$
$0 \neq 8e^{4x}$

$y_p = Axe^{4x}$
$y' = A[x \times 4e^{4x} + e^{4x}(1)]$
$y' = A[4xe^{4x} + e^{4x}]$
$y' = 4Axe^{4x} + Ae^{4x}$
$y'' = A[4x(4e^{4x}) + e^{4x}(4) + 4e^{4x}]$
$y'' = A[16xe^{4x} + 4e^{4x} + 4e^{4x}]$
$y'' = A[16xe^{4x} + 8e^{4x}]$
$y'' = 16Axe^{4x} + 8Ae^{4x}$
Substitute into $y'' - 6y' + 8y = 8e^{4x}$
$16Axe^{4x} + 8Ae^{4x} - 6(4Axe^{4x} + Ae^{4x}) + 8(Axe^{4x}) = 8e^{4x}$
$\cancel{16Axe^{4x}} + 8Ae^{4x} - \cancel{24Axe^{4x}} - 6Ae^{4x} + \cancel{8Axe^{4x}} = 8e^{4x}$

$8Ae^{4x} - 6Ae^{4x} = 8e^{4x}$
$\cancel{2}Ae^{4x} = \cancel{8}e^{4x}$
$A = 4$

$y = C_1e^{4x} + C_2e^{2x} + 4xe^{4x}$

Q.E.D.