Iwunze Favour
15/SCI09/002.

Assignment 2

i) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1, \quad b = 4; \quad c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 + \sqrt{-4}}{2}$

$m = \frac{-4 + \sqrt{-1} \times \sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$\alpha = -2 \quad \beta = 1$

CF: $y = e^{\alpha\theta}(A\cos\beta\theta + B\sin\beta\theta)$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

PI: $y = C\cos\theta + D\sin\theta$

$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\sin\theta)$
$+5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta$
$+5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C\cos\theta + 4D\cos\theta + 5C\cos\theta = 0$

$4D\cos\theta + 4C\cos\theta = 0$

$4D + 4C = 0$

$4C = -4D$

$C = -D$

$-D\sin\theta - 4C\sin\theta + 5D\sin\theta = 6\sin\theta$

$-4C\sin\theta + 4D\sin\theta = 6\sin\theta$

$-4C + 4D = 6$

$\therefore -4(-D) + 4D = 6$

$4D + 4D = 6$

$8D = 6$

$D = 6/8 = 3/4$

$C = -D$

$\therefore C = -3/4$

PI: $y = -3/4\cos\theta + 3/4\sin\theta$

GS = CF + PI

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

ii) $y = -3/4\cos\theta + 3/4\sin\theta$

$y = -0.75\cos\theta + 0.75\sin\theta$

iii) at steady state $\frac{dy}{dx} = 0$

$\theta = \infty$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta$
$+ 3/4\sin\theta$

$\frac{dy}{d\theta} = Vdu + Udv\ (3/4\cos\theta + 3/4\sin\theta)$

$u = e^{-2\theta}$

$du = -2e^{-2\theta}$

$v = A\cos\theta + B\sin\theta$

$dv = -A\sin\theta + B\cos\theta$

$\frac{dy}{dy} = -2e^{-2\theta}(A\cos\theta + B\sin\theta) + e^{-2\theta}$
$(-A\sin\theta + B\cos\theta) + 3/4\sin\theta + 3/4$

At Steady State, $\theta = \infty$ & $\frac{dy}{d\theta} = 0$

$e^{-2\theta} = e^{-2\infty} = 0$

$\frac{dy}{d\theta} = 0$

$0 = 3/4 \sin\theta + 3/4 \cos\theta$

$3/4 \sin\theta = -3/4 \cos\theta$

$\frac{3/4 \sin\theta}{3/4 \cos\theta} = \frac{-3/4 \cos\theta}{3/4 \cos\theta}$

$\frac{\sin\theta}{\cos\theta} = -1$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45°$

2.) $EI \frac{d^2y}{dx^2} = \frac{\omega}{2}(L-x)^2$

Solution

$EI \frac{d^2y}{dx^2} = \frac{\omega}{2}(l^2 - 2xl + x^2)$

Assumed PI $y = Px^2 + Qx^3 + Rx^4 \dots (i)$

$y' = 2Px + 3Qx^2 + 4Rx^3 \; (ii)$

$y'' = 2P + 6Qx + 12Rx^2 \dots (iii)$

Substitute the eqn (iii) into the original eqn

$EI(2P + 6Qx + 12Rx^2) = \frac{\omega}{2}(l^2 - 2xl + x^2)$

Comparing coefficient

$\frac{dy}{dx} = B + \frac{\omega}{24EI}(12l^2x - 12lx^2 + 4x^3)$

$\frac{dy}{dx} = B + \frac{4\omega}{24EI}x(3l^2 - 3lx + x^2) \dots (*)$

For eqn * $y = 0$, $x = 0$.

$0 = A + \frac{\omega}{24EI}(0)(6L^2 - 4lx + x^2)$

$A = 0$

from eqn * $dy/dx = 0$, $x = 0$

$0 = B + 0$

$B = 0$

GS! $y = \frac{\omega}{24EI}x^2(6l^2 - 4lx + x^2)$

When $x = L$

PS it becomes

$y = \frac{\omega}{24EI}(L)^2(6L^2 - 4l^2 + l^2)$

$y = \frac{\omega(l)^2}{24EI}(3l^2)$

$y = \frac{3\omega l^4}{24EI}$

$y = \frac{\omega L^4}{8EI}$

TABLE OF Y AGAINST θ

θ	Y
0	-0.75
30	-0.274519052838329
60	0.274519052838329
90	0.75
120	1.02451905283833
150	1.02451905283833
180	0.75
210	0.274519052838329
240	-0.274519052838328
270	-0.75