AGBOOLA AMOS A
15/ENG04/005
ELECT/ELECT ENGR

1) $d^2y/d\theta^2 + 4\,dy/d\theta + 5y = 6\sin\theta$

C.F $m^2+4m-5=0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2-20}}{2(1)} = \frac{-4 \pm \sqrt{16-20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm 2j}{2}$

$= -2 \pm j$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

Assumed PI $y = C\cos\theta + D\sin\theta$

$dy/d\theta = -C\cos\theta + D\cos\theta$

$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$

$-\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

comparing coefficients

$\sin\theta: -D - 4C + 5D = 6$

$\cos\theta: -C + 4D + 5C = 0$

$4D - 4C = 6$

$4D + 4C = 0$

$-8C = 6$

$C = -6/8$

$C = -3/4$

$4(-3/4) + 4D = 0$

$D = 3/4$

Assumed P.I $y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

G.S $\Rightarrow$ C.F + P.I

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

ii) $y = \frac{3}{4}(\sin\theta - \cos\theta)$

for $\theta = 0$ to 270°

P.I $y = \frac{3}{4}(\sin\theta - \cos\theta)$

$\frac{dy}{d\theta} = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

$\frac{dy}{d\theta} = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

at steady state

$\frac{dy}{d\theta} = 0$ and $\theta = \infty$

$0 = \frac{3}{4}(\cos\theta + \sin\theta)$

$-\cos\theta = \sin\theta$

divide through by $\cos\theta$

$\frac{-\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45^\circ = 315^\circ$

2) $EI\frac{d^2y}{dx^2} = \frac{w}{2}(l-x)^2$

C.f Auxiliary equation

$m^2 = 0$

$m = \sqrt{0}$

$m = \pm 0$

$y = e^0(A + Bx)$

$y = A + Bx$

Assumed P.I

$y = Cx^2 + Dx^3 + fx^4$ — (i)

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4fx^3$ — (2)

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12fx^2$ — (3)

cutting eqn 3 into orginal equation

$EI(2C+6Dx+12fx^2) = w/2(l-x)^2$

$2CEI+6DEIx+12EfIx^2 = w/2(l-x)^2$

$2CEI+6DEIx+12EfIx^2 = w/2(l^2-2lx+x^2)$

$4CEI+12DEIx+24EfIx^2 = w(l^2-2lx+x^2)$

$4CEI+12DEIx+24EfIx^2 = wl^2-2wlx+wx^2$

comparing coefficient

$x^2: 24EfI = 2w$

$f = \frac{w}{24EI}$

$x: 12DEI = -2wl$

$D = \frac{-2wl}{12EI}$

constant: $4CEI = wc^2$

$C = \frac{wc^2}{4EI}$

Subt back into orginal equation CPI

$y = \frac{wl^2}{4EI}x^2 + \frac{-2wl}{12EI} + \frac{w}{24EI}x^4$

G.S $\Rightarrow y = Cf + PI$

$= A+Bx + \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$

$\frac{dy}{dx} = B + \frac{2wl^2}{4EI}x - \frac{6wl}{12EI}x^2 + \frac{8w}{24EI}x^3$

at $y=0$ and $x=0$

$y = A+Bx + \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^2 + \frac{2w}{24EI}x^4$

$0 = A$

at $dy/dx = 0$, and $x=0$

$0 = B + \frac{2wl^2}{4EI}x - \frac{6wlx^2}{12EI} + \frac{8^2w}{24EI}x^3$

$$0 = B + \frac{wl^2}{2EI} - \frac{wl^2}{2EI}x^2 + \frac{w}{3EI}x^3$$

$$B = 0$$

G.S

$$y = \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$$

$$y = \frac{wl^2}{4EI}x^2 - \frac{wl}{6EI}x^3 + \frac{w}{24EI}x^4$$

at $x = l$

$$y = \frac{wl^4}{4EI} - \frac{wl^4}{6EI} + \frac{w^4}{24EI}$$

$$y = \frac{6wl^4 - 4wl^4 + wl^4}{24EI}$$

$$y = \frac{3wl^4}{24EI}$$

$$y = \frac{wl^4}{8EI}$$