15/EN404/003
ELECTRICAL ELECTRONICS

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$$

Solution

$$m^2 + 4m + 5 = 0$$

Using $m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$m = \frac{-4 \pm \sqrt{4}\cdot\sqrt{1}}{2}$$

$$m = \frac{-4}{2} \pm \frac{2j}{2}$$

$$m = 2 \pm j$$

$$y = e^{2\theta}[(\cos\theta + B\sin\theta]$$

PI

$$y = A\cos\theta + B\sin\theta$$

$$dy/d\theta = -A\sin\theta + B\cos\theta$$

$$\frac{d^2y}{d\theta^2} = -A\cos\theta - B\sin\theta$$

$$\therefore [-A\cos\theta - B\sin\theta] + 4[-A\sin\theta + B\cos\theta] + 5[A\cos\theta + B\sin\theta] = 6\sin\theta$$

$$-A\cos\theta - B\sin\theta - 4A\sin\theta - 4B\cos\theta + 5A\cos\theta + 5A\sin\theta = 6\sin\theta$$

$$-A\cos\theta + 4B\cos\theta + 5A\cos\theta - B\sin\theta - 4A\sin\theta + 5B\sin\theta = 6\sin\theta$$

Comparing coefficient

$$-A + 4B + 5A = 0$$

$$4A + 4B = 0 \quad \text{——— eqn 1}$$

$$-B + 4A + 5B = 6$$

$$-4A + 4B = 6 \quad \text{——— eqn 2}$$

add eqn 1 and 2

$$8B = 6$$

$$B = 3/4$$

$$4A + 4(3/4) = 0$$

$$4A = -3 \quad ; \quad A = -3/4$$

$y = -3/4 \cos\theta + 3/4 \sin\theta$ — PI

$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

at $\theta = \infty$ and $\frac{dy}{dx} = 0$

~~$\frac{dy}{dx} = e^{-2\theta}[-(3\sin\theta + D\cos\theta) - 3/4\cos\theta + 3/4\sin\theta$~~

$\frac{dy}{d\theta} = e^{-2\theta}[-(3\sin\theta + D\cos\theta) + [C\cos\theta + D\sin\theta]$

$-2e^{-2\theta} + 3/4\sin\theta + 3/4\cos\theta$

at $\theta = \alpha$ and $\frac{dy}{dx} = 0$

$\theta = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$-\cos\theta = \sin\theta$

D.B.J by $-\cos\theta$

$\frac{-\cos\theta}{-\cos\theta} = \frac{\sin\theta}{-\cos\theta}$

$\frac{\sin\theta}{-\cos\theta} = 1$

$-\tan\theta = 1$

$\tan\theta = -1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

2. $EI \frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

C.F

Auxiliary equation

$m^2 = 0$

$m = \sqrt{0}$

$m = \pm 0$ twice

$y = e^0 (A + Bx)$

$y = A + Bx$

Assume P.I

$y = (x^2 + Dx^3 + Fx^4)$

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Fx^3$

$\frac{d^2y}{dx^3} = 2C + 6Dx + 12Fx^2$

$EI(2C + 6Dx + 12Fx^2) = \frac{w}{2}(L-x)^2$

$2CEI - 6DEIx + 12FCIx^2 = \frac{w}{2}(L-x)^2$

$2(CEI - 6DEIx + 12FEIx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4CEI + 12DEIx + 24FEIx^2 = W(L^2 - 2Lx + x^2)$

$4CEI + 12DEIx + 24FEIx^2 = WL^2 - 2Lx + Wx^2$

Comparing coefficient

$x^2 \div 24FEI = W$

$F = \frac{W}{24EI}$

$-12DEI = -2WL$

$D = \frac{-WL}{6EI}$

Constant: $4CEI = WL^2$

$C = \frac{WL^2}{4EI}$

$\therefore y = \frac{WL^2}{4EI}x^2 - \frac{WLx^3}{6EI} + \frac{W}{24EI}x^2$

$\therefore$ G.S $= y = A + Bx + \frac{Wl^2}{4EI}x^2 - \frac{Wl}{6EI}x^3 + \frac{W}{24EI}x^4$

$\frac{dy}{dx} = B + \frac{Wl^2}{2EI}x - \frac{Wl}{2EI}x^2 + \frac{W}{6EI}x^3$

at $\frac{dy}{dx} = 0, \quad x = 0$

$0 = B$

at $y = 0, \quad x = 0.$

$A = 0$

G.S

$y = \frac{Wl^2}{4EI}x^2 - \frac{Wl}{6EI}x^3 + \frac{W}{24EI}x^4$

at $x = l$

$y = \frac{Wl^4}{4EI} - \frac{Wl^4}{6EI} + \frac{Wl^4}{24EI}$

$y = \frac{6Wl^4 - 4Wl^4 + Wl^4}{24EI}$

$y = \frac{3Wl^4}{24EI}$

$y = \frac{Wl^4}{8EI}$