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ELECTRICAL/ELECTRONICS
ENG 281

The Parametric equation of a curve are given in equation (i) & (ii)

$$x = \cos t + t\sin t \quad \text{— (i)}$$
$$y = \sin t - t\cos t \quad \text{— (ii)}$$

In terms of t, determine
(a) An expression for the radius of curvature R and
(b) expressions for the co-ordinates (h, k) of the centre of curvature.

Solution

$$x = \cos t + t\sin t$$
$$y = \sin t - t\cos t$$

$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t + \sin t$$

using Products rule

$u = t \qquad v = \sin t$

$\frac{du}{dt} = 1 \qquad \frac{dv}{dt} = \cos t$

$$u\frac{dv}{dt} + v\frac{du}{dt}$$
$$= t(\cos t) + \sin t(1)$$
$$= t\cos t + \sin t$$

$$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$$

using Product rule

$u = -t \qquad v = \cos t$

$\frac{du}{dt} = -1 \qquad \frac{dv}{dt} = -\sin t$

$$v\frac{du}{dt} + u\frac{dv}{dt}$$
$$= -t(-\sin t) + \cos t(-1)$$
$$= t\sin t - \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$dy/dx = \frac{t\sin t}{t\cos t}$$
$$= \tan t$$
$$\frac{d^2y}{dx^2} = \sec^2 t$$

$$R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$

$$R = \frac{\left[1 + (\tan t)^2\right]^{3/2}}{[\sec^2 t]}$$

$$h = x - R\sin\theta$$
$$k = y + R\cos\theta$$

$$\tan t = \frac{dy}{dx}$$

$$h = [\cos t + t\sin t] - \left[\frac{(1 + [\tan t]^2)^{3/2}}{[\sec^2 t]}\right]\sin[\tan^{-1}[\tan t]]$$

$$k = [\sin t - t\cos t] + \left[\frac{[1 + [\tan t]^2]^{3/2}}{(\sec^2 t}\right]\cos[\tan^{-1}(\tan t)]$$