

Agha-Ibrahim Uchehams
Mechanical Engg
15/2/2018
Eita 351

$$d) \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin x$$

convert to homogeneous equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$a = 1, b = 4, c = 5$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$m = \frac{-4 \pm 2j}{2}$$

$$m = -2 \pm j$$

$$C.F = y = e^{-2x} (A \cos x + B \sin x)$$

$$y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$-C \cos x - D \sin x + 4[-C \sin x + D \cos x] + 5[C \cos x + D \sin x] = 6 \sin x$$

$$-C \cos x - D \sin x - 4C \sin x + 4D \cos x + 5C \cos x + 5D \sin x = 6 \sin x$$

$$-C + 4D + 5C = 0$$

$$-D + 4C + 5D = 6$$

$$4C + 4D = 0 \quad \dots \textcircled{I}$$

$$-4C + 4D = 6 \quad \dots \textcircled{II}$$

$$8D = 6$$

$$D = \frac{3}{4}$$

sub D in eqn ①

$$-4C + 4\left(\frac{3}{4}\right) = 6$$

$$-4C + 3 = 6$$

$$-4C = 3$$

$$C = -\frac{3}{4}$$

$$y = -\frac{3}{4} \cos x + \frac{1}{4} \sin x$$

$$y = e^{-2x} (A \cos x + B \sin x) + \frac{1}{4} \cos x + \frac{1}{4} \sin x$$

$$y = e^{-2x} (A \cos x + B \sin x) + \frac{1}{4} (\cos x - \sin x)$$

at steady state

$$\frac{dy}{dx} = 0 \quad \text{at } x = \infty$$

$$y = e^{-2x} (A \cos x + B \sin x) + \frac{1}{4} (\cos x - \sin x)$$

$$\frac{dy}{dx} = e^{-2x} (B \cos x - A \sin x) - 2e^{-2x} (A \cos x + B \sin x) + \frac{1}{4} (\sin x + \cos x)$$

$$\frac{dy}{dx} = e^{-2x} (A \cos x - A \sin x) - 2e^{-2x} (A \cos x + B \sin x) + \frac{1}{4} (\sin x + \cos x)$$

$$\frac{dy}{dx} = \frac{1}{4} (\sin x - \cos x)$$

$$2) \quad \mathcal{E} \left[\frac{dy}{dx} \right] = \frac{1}{2} (1-x)^2$$

$$\mathcal{E} \left[\frac{dy}{dx} \right] = 0$$

$$\mathcal{E} [m] = 0$$

$$m^2 = 0 \Rightarrow m = \pm \sqrt{0} = 0$$

$$m_1 = m_2 = 0$$

$$y = e^{0x} (A + Bx)$$

$$CF \quad y = A + Bx$$

$$y = Px^2 + Sx + Tx^3$$

$$\frac{dy}{dx} = 2Px + S + 3Tx^2$$

$$\frac{d^2y}{dx^2} = 2P + 6Tx$$

$$\mathcal{E} [2P + 6Sx + 12Tx^2] = \frac{1}{2} (1-x)^2$$

$$2PE + 6S \cdot E + 12T \cdot E = \frac{1}{2} [1 - 2x + x^2]$$

$$4PE + 12S \cdot E + 24T \cdot E = \frac{1}{2} [1 - 2x + x^2]$$

$$24TEI = \omega$$

$$T = \frac{\omega}{24EI}$$

$$12SEI = -2\omega l$$

$$S = \frac{-2\omega l}{24EI}$$

$$y = \left[\frac{\omega l^2}{-4EI} \right] x^2 - \left[\frac{\omega l}{6EI} \right] x^3 + \left[\frac{\omega}{24EI} \right] x^4$$

$$y = \frac{\omega l^2 x^2}{4EI} - \frac{\omega l x^3}{6EI} + \frac{\omega x^4}{24EI}$$

$$y = \frac{6\omega l^2 x^2}{24EI} - \frac{4\omega l x^3}{24EI} + \frac{\omega x^4}{24EI}$$

$$y = \frac{6\omega l^2 x^2}{24EI} - \frac{4\omega l x^3}{24EI} + \frac{\omega x^4}{24EI}$$

$$PI: y = \frac{\omega}{24EI} [6l^2 x^2 - 4lx^3 + x^4]$$

$$y = A + Bx + \frac{\omega}{24EI} [6l^2 x^2 - 4lx^3 + x^4]$$

$$\text{at } x=0, y=0, \frac{dy}{dx} = 0$$

$$0 = A + B(0) + \frac{\omega}{24EI} [12l^2 x - 12lx^2 + 4x^3]$$

$$0 = B + \frac{\omega}{24EI} [12l^2(0) - 12l(0)^2 + 4(0)^3]$$

$$B = 0$$

$$\text{when } A = B = 0$$

$$y = 0 + 0x + \frac{\omega}{24EI} [6l^2 x^2 - 4lx^3 + x^4]$$

$$y = \frac{\omega}{24EI} [6l^2 x^2 - 4lx^3 + x^4]$$

$$\text{when } x=L$$

$$y = \frac{\omega}{24EI} [6l^4 - 4l^4 + l^4]$$

$$y = \frac{w}{24EI} [3l^4]$$

$$y = \frac{w l^4}{8EI}$$

