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15/ENG01/003
CHEMICAL ENGINEERING

1 $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

Solution

$m^2 + 4m + 5 = 0$

using quadratic formula

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = 1, b = 4, c = 5$

$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{(4)^2 - 20}}{2}$

$= \frac{-4 \pm \sqrt{16-20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm 2\sqrt{-1}}{2}$

$= \frac{4 \pm 2j}{2}$

$= -2 \pm j$

C.F: $y = e^{-2\theta}[C\cos\theta + D\sin\theta]$

P.I: $y = A\cos\theta + B\sin\theta$

$\frac{dy}{d\theta} = -A\sin\theta + B\cos\theta$

$\frac{d^2y}{d\theta^2} = -A\cos\theta - B\sin\theta$

substituting into the given equation

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

collecting like terms,

$-A\cos\theta + 4B\cos\theta + 5A\cos\theta - B\sin\theta - 4A\sin\theta + 5B\sin\theta = 6\sin\theta$

factorising

$\cos\theta[-A + 4B + 5A] + \sin\theta[-B - 4A + 5B] = 6\sin\theta$

$\cos\theta[4B + 4A] + \sin\theta[4B - 4A]$

$4B + 4A = 0$ --- ①

$4B - 4A = 6$ --- ②

From eqn ①,

$4B = -4A$

$B = \frac{-4A}{4}$

$B = -A$ --- ③

Substituting eqn ③ into ②

$4B - 4A = 6$

$4(-A) - 4A = 6$

$-4A - 4A = 6$

$-8A = 6$

$A = 6/-8$

$A = -3/4$

Since $B = -A$

$B = -(-3/4)$

$B = 3/4$

P.I : $y = -3/4\cos\theta + 3/4\sin\theta$

G.S : $y = e^{-2\theta}[C\cos\theta + D\sin\theta] - 3/4\cos\theta + 3/4\sin\theta$

2. $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

Solution

$EIm^2 = 0$

$m^2 = 0$

$m = \pm 0$

$y = e^{0x}[A+Bx]$

$y = A + Bx$

P.I. $y = Fx^2 + Gx^3 + Hx^4$

$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$

$\frac{d^2y}{dx^2} = 2F + 6Gx + 12Hx^2$

substituting into the given equation

$EI[2F + 6Gx + 12Hx^2] = \frac{w}{2}[L-x]^2$

$2FEI + 6GEIx + 12HEIx^2 = \frac{w}{2}[L-x]^2$

Cross-multiply

$4FEI + 12GEIx + 24HEIx^2 = w[L-x]^2$

$4FEI + 12GEIx + 24HEIx^2 = w[L^2 - 2Lx + x^2]$

$24HEI = w$

$H = \frac{w}{24EI}$ ---(1)

$12GEI = -2wL$

$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$ ---(2)

$4FEI = wL^2$

$F = \frac{wL^2}{4EI}$

$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$

$= \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$

$$= \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$G.S: y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 + 4Lx^3 + x^4\right] -$$

at $y = 0$, $x = 0$, $dy/dx = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left[6L^2(0) - 4L(0) + 0\right]$$

$$0 = A$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{w}{24EI}\left[12(0) - 12(0) + 4(0)\right]$$

$$B = 0$$

$$P.S: y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$y = \frac{wx^2}{24EI}\left[x^2 - 4Lx + 6L^2\right]$$

when $x = L$

$$y = \frac{wL^2}{24EI}\left[L^2 - 4L^2 + 6L^2\right]$$

$$= \frac{wL^2}{24EI}\left[3L^2\right]$$

$$= \frac{wL^4}{8EI}$$

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