Ogbapu Emmanuel Umofon
PS/ENG/08/041
ENG 381
ELECT/ELECT

(1) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$

(i) C.F

$m^2 + 4m + 5 = 0 \quad a = 1, b = 4, c = 5$

using $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm j\sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$\therefore y = e^{-2x}(A\cos x + B\sin x)$

P.I.

$f(x) = 6\sin\theta$

$\therefore y = C\cos\theta + D\sin\theta$

$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta)$
$+ 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta +$
$5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$4C\cos\theta + 4D\sin\theta + 4C\sin\theta + 4D\sin\theta$
$= 6\sin\theta$

$4D - 4C = 6$ —— (i)

$+4C + 4D = 6$

$4D = 6 + 4C$

$D = \frac{6}{4} + C$

$4(6/4 + C) + 4C = 0$

$6 + 4C + 4C = 0$

$6 + 8C = 0$

$8C = -6$

$C = -6/8$

$C = -3/4$

$D = 6/4 - 3/4 = 3/4$

$C = -3/4, \; D = 3/4$

$\therefore$ The PI $= y = -3/4\cos\theta + 3/4\sin\theta$

G.S = PI + CF

$y = e^{-2x}(A\cos x + B\sin x) - 3/4\cos\theta + 3/4\sin\theta$

at steady state

$y_P \Rightarrow -3/4\cos\theta + 3/4\sin\theta$

$y_P \Rightarrow 3/4\sin\theta + 3/4\cos\theta$

$3/4\sin\theta = -3/4\cos\theta$

$\sin\theta = -\cos\theta$

Div by $\cos\theta$, $\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}$

$\tan\theta = -1$

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

$$EI\,m^2 = 0$$

$$m^2 = 0$$

$$m = \pm\sqrt{0}$$

$$y = e^{0x}[A+Bx]$$

$$\therefore y = A + Bx$$

$$y_p = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$$

$$\frac{d^2y}{dx^2} = 2F + 6Gx + 12Hx^2$$

$$EI[2F + 6Gx + 12Hx^2] = \frac{w}{2}(L-x)^2$$

$$2FEI + 6GxEI + 12Hx^2EI = \frac{w}{2}(L-x)^2$$

$$4FEI + 12GxEI + 24Hx^2EI = w(L-x)^2$$

$$24HEI = w$$

$$H = \frac{w}{24EI}$$

$$12GEI = -2wL$$

$$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \quad \ldots (11)$$

$$4FEI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

From

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$= \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$= \cancel{wL^2x^2}$$

$$= \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$G.S = y = A + Bx + \frac{w}{24EI}(6L^2x^2 - 4Lx^3 + x^4)$$

at $y = 0$, $x = 0$, $dy/dx = 0$

$$0 = A + B(0) + \frac{w}{24EI}[6L^2(0) - 4L(0) + 0]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}[12L^2 - 12(x) + 4(x)$$

$$0 = B + \frac{w}{24EI}[12L^2 - 12(0) + 4(0)]$$

$$B = 0$$

$$y_p = \frac{wx^2}{24EI}[6L^2x^2 - 4Lx^3 + x^4]$$

$$y_p = \frac{wx^2}{24EI}[6L^2 - 4xL + x^2]$$

$$y_p = \frac{wL^2}{24EI}[x^2 - 4xL + 6L^2]$$

when $x = L$

$$y_p = \frac{wL^2}{24EI}[3L^2]$$