ASSIGNMENT 2

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15ENG07/010

Petroleum Engineering

1.) $\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$

// Convert equation into an homogenous equation

$\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1, \ b = 4, \ c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1} \Rightarrow m = \frac{-4 \pm \sqrt{-4}}{2} \Rightarrow m = \frac{-4 \pm 2i}{2}$

$m = -2 \pm i$

$m_1 = -2 + i \ ; \ m_2 = -2 - i$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$ // C.F.

$y = C\cos\theta + D\sin\theta$

$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$

$-\cos\theta - D\sin\theta + 4[-C\sin\theta + D\cos\theta] + 5[C\cos\theta + D\sin\theta] = 6\sin\theta$

$-\cos\theta - D\sin\theta - 4C\sin\theta + 4D\sin\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C + 4D + 5C = 0$

$-D + 4C + 5D = 6$

$4C + 4D = 0$ ——— (1)

$-4C + 4D = 6$ ——— (2)

$8D = 6$

$D = \frac{6}{8} = \frac{3}{4}$

// substitute $D = \frac{3}{4}$ into equation (2)

$4 \times \frac{3}{4} - 4C = 6$

$4C = 3 - 6 = -3, \quad C = -\frac{3}{4}$

$y = e^{-2t}(A\cos t + B\sin t) + 3/4\sin t - 3/4\cos t$

(ii) $y = 3/4(\sin t - \cos t)$

for $t = 0$ to $210°$

Considering D.T

$y = 3/4(\sin t - \cos t)$

$\frac{dy}{dt} = 3/4\sin t - 3/4\cos t$

$\frac{dy}{dt} = 3/4\cos t + 3/4\sin t$

at steady state

$\frac{dy}{dt} = 0$ and $t = \infty$

$0 = 3/4(\cos t + \sin t)$

$-\cos t = \sin t$

divide through by $\cos t$

$\frac{-\cos t}{\cos t} = \frac{\sin t}{\cos t}$

$\tan t = -1$

$t = \tan^{-1}(-1)$

$t = -45$

$t = 315°$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\sin\theta - 3/4\cos\theta$

(ii) $y = 3/4(\sin\theta - \cos\theta)$

for $\theta = 0$ to $210°$

Considering D.T

$y = 3/4(\sin\theta - \cos\theta)$

$\frac{dy}{d\theta} = 3/4\sin\theta - 3/4\cos\theta$

$\frac{dy}{d\theta} = 3/4\cos\theta + 3/4\sin\theta$

at steady state

$\frac{dy}{d\theta} = 0$ and $\theta = \theta$

$0 = 3/4(\cos\theta + \sin\theta)$

$-\cos\theta = \sin\theta$

divide through by $\cos\theta$

$\frac{-\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45$

$\theta = 315°$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $x = 0$, $y = 0$ $\quad \frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left[6L^2(0)^2 - 4L(0)^3 + (0)^4\right]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left[12L^2x^2 - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{w}{24EI}\left[12L^2(0) - 12L(0)^2 + 4(0)^3\right.$$

$$B = 0$$

when $A = B = 0$

$$y = 0 + 0x + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

when $x = L$

$$y = \frac{w}{24EI}\left[6L^4 - 4L^4 + L^4\right.$$

$$y = \frac{w}{24EI}\left[3L^4\right]$$

$$y = \frac{wL^4}{8EI}$$

$$\frac{d^2y}{dx^2} = 2R + 6Sx + 12Tx^2$$

$$EI\left[2R + 6Sx + 12Tx^2\right] = \frac{w}{2}(L-x)^2$$

$$2REI + 6SxEI + 12Tx^2EI = \frac{w}{2}\left[L^2 - 2Lx + x^2\right]$$

// multiply eqn by 2

$$4REI + 12SxEI + 24Tx^2EI = wL^2 - 2wLx + wx^2$$

$$24TEI = w$$

$$T = \frac{w}{24EI}$$

$$12SEI = -2wL$$

$$S = \frac{-2wL}{24EI}$$

$$4REI = wL^2$$

$$R = \frac{wL^2}{4EI}$$

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$