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15/ENG06/059
Mechanical Engineering
ENG351 (2)

(1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

CF $\frac{dy}{dx} = m$

$m^2 + 4m + 5 = 0$

$\cancel{m^2 + 5m - m + 5 = 0}$ $\qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$

$\cancel{(m+5)(m-1) = 0}$

$\frac{-b}{2} \pm \frac{\sqrt{b^2 - 4ac}}{2}$

$\frac{-4}{2} \pm \frac{\sqrt{4^2 - 4(1)(5)}}{2} = -2 \pm \frac{\sqrt{16 - 20}}{2}$

$-2 \pm \frac{\sqrt{-4}}{2} = -2 \pm \frac{2i}{2}$

$= -2 \pm i$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

PF

Assumed PI: $y = C\cos\theta + D\sin\theta$ — (i)

$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$ — (ii)

$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$ — iii

subst. (i), (ii) (iii) into the original eqn

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C\cos\theta + 5C\cos\theta + 4D\cos\theta - 4C\sin\theta - D\sin\theta + 5D\sin\theta = 6\sin\theta$

$4C\cos\theta + 4D\cos\theta - 4C\sin\theta + 4D\sin\theta = 6\sin\theta$

Comparing coefficient.

$4C + 4D = 0$

$4C = -4D$

$C = -D$

$-4C + 4D = 6$

subst- $C = -D$ into the above eqn

$-4(-D) + 4D = 6$

$4D + 4D = 6$

$8D = 6$

$D = 6/8 = 3/4$

$\therefore C = -3/4$

so PF : $y = -3/4 \cos\theta + 3/4 \sin\theta \simeq -3/4 (\cos\theta - \sin\theta)$

The general soln.

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4(\cos\theta - \sin\theta)$

(iii) At stationary $dy/dx = 0$

$\frac{dy}{dx} = e^{-2\theta}(-A\sin\theta + B\cos\theta) - 2e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4(-\sin\theta - \cos\theta)$

$\frac{dy}{dx} = e^{-2\theta}(-A\sin\theta + B\cos\theta) - 2e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\sin\theta + \cos\theta)$

let the constants, $A = 0$, $B = 0$ & $\frac{dy}{dx} = 0$

$0 = 3/4(\sin\theta + \cos\theta)$

$0 = \sin\theta + \cos\theta$

$\sin\theta = -\cos\theta$

$\frac{\sin\theta}{\cos\theta} = -1$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45° \simeq 135°$

2. $EI\frac{d^2y}{dx^2} = \frac{w}{2}(l-x)^2$

soln

$EI\frac{d^2y}{dx^2} = \frac{w}{2}(l^2 - 2xl + x^2)$

CF

$EI\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} = 0$

$m^2 = \pm\sqrt{0}$

$m = \sqrt{0}$

$y = e^0(A + Bx)$

$y = A + Bx$

~~PF~~

~~Assumed PI $y = Cx + D + Cx^2 + Dx + E$ (i)~~

~~$\frac{dy}{dx} = C + Cx + D$ — (ii)~~

~~$\frac{d^2y}{dx^2} = C$ — (iii)~~

~~Subst eqn (iii) into the original equation~~

~~$EIC = \frac{w}{2}(l^2 - 2xl + x^2)$~~

PF

Assumed PI: $y = Px^2 + Qx^3 + Rx^4$ — (i)

$y' = 2Px + 3Qx^2 + 4Rx^3$ — (ii)

$y'' = 2P + 6Qx + 12Rx^2$ — (iii)

Substitute the eqn (iii) into the original eqn

$EI(2P + 6Qx + 12Rx^2) = \frac{w}{2}(l^2 - 2xl + x^2)$

Comparing Coefficient

$EI\,2P = \frac{wl^2}{2}$

$P = \frac{wl^2}{4EI}$

$EI\,6Q = \frac{-2wl}{2} = -wl$

$Q = \cancel{\frac{-wl}{EI}} \quad \frac{-wl}{6EI}$

$EI\,12R = \frac{w}{2}$

$R = \frac{w}{24EI}$

PF: $y = \frac{wl^2}{4EI}x^2 + \left(\frac{-wl}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)x^4$

$\cancel{y = \frac{24w}{EI}\left(l^2\right.}$

$\cancel{y = \frac{24w}{EI}}\left(6l^2\right.$

$y = \frac{wx^2}{2EI}\left[\frac{l^2}{2} - \frac{l}{3}x + \frac{1}{12}x^2\right]$

$y = \frac{w}{2EI}x^2\left[\frac{6l^2 - 4lx + x^2}{12}\right)$

$y = \frac{w}{24EI}x^2\left[6l^2 - 4lx + x^2\right)$

G.S

$y = A + Bx + \frac{w}{24EI}x^2\left(6l^2 - 4lx + x^2\right) \rightarrow (*)$

at $y = 0$, $x = 0$, $dy/dx = 0$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left(12l^2x - 12lx^2 + 4x^3\right)$$

$$\frac{dy}{dx} = B + \frac{4w}{24EI}\, x\left(3l^2 - 3lx + x^2\right) \quad -(**)$$

For eqn (*) $y=0$, $x=0$

$$0 = A + \frac{w}{24EI}(0)(6l^2 - 4lx + x^2$$

$$A = 0$$

From eqn ** $dy/dx = 0$, $x = 0$

$$0 = B + 0$$

$$B = 0$$

The general soln becomes

$$y = \frac{w}{24EI}\, x^2\left(6l^2 - 4lx + x^2\right)$$

when $x = l$

It becomes.

$$y = \frac{w}{24EI}\,(l)^2\left(6l^2 - 4l^2 + l^2\right)$$

$$y = \frac{w(l)^2}{24EI}\left(3l^2\right)$$

$$y = \frac{3wl^4}{24EI}$$

$$y = \frac{wl^4}{8EI}$$