ASSIGNMENT 2.

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(1) The parametric equ. of a curve are as given in equ (1) and (2)

$$x = \cos t + t\sin t \quad \text{---- (1)}$$
$$y = \sin t - t\cos t \quad \text{---- (2)}$$

In terms of t, determine
(a) An expression for the radius of curvature (R), and
(b) Expressions for the coordinates (h, k) of the centre of curvature.

Solution.

(a) $y = \sin t - t\cos t$

$x = \cos t + t\sin t$

$$\frac{dy}{dt} \Rightarrow \cos t - (-t\sin t + \cos t)$$
$$\Rightarrow \cos t + t\sin t - \cos t$$
$$\Rightarrow \cos t - \cos t + t\sin t \qquad \text{Recall:- } \cos t + \sin t = 1$$
$$\therefore \frac{dy}{dt} = t\sin t$$

$$\frac{dx}{dt} \Rightarrow -\sin t + (t\cos t + \sin t)$$
$$\Rightarrow -\sin t + t\cos t + \sin t$$
$$\Rightarrow t\cos t - \sin t + \sin t$$
$$\frac{dx}{dt} = t\cos t \text{ Ans.}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t\sin t}{t\cos t} = \tan t \text{ Ans.}$$

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t\cos t} = \frac{1}{t} \times \frac{1}{\cos^3 t} = t^{-1}\sec^3 t$$

$$\frac{d^2y}{dx^2} = t^{-1}\sec^4 t$$

$$R = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left(1+(\tan t)^2\right)^{3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(1+\tan^2 t)^{3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(\sec t)^{2\times 3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(\sec t)^3}{t^{-1}(\sec t)^3} = \frac{1}{1} \div \frac{1}{t}$$

$$\therefore R = t$$

$$\therefore R = t \text{ units}$$

(b) ~~Expression for~~ Centre of Curvature:-

$$x = h + R\sin\theta$$

$$h = x - R\sin\theta$$

$$k = y + R\cos\theta$$

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

$$\theta = \tan^{-1}(\tan t)$$

$$\theta = t$$

$$x_1 = \cos t + t\sin t$$

$$h = \cos t + (\sin t - (t)\sin t$$

$$h = \cos t \cancel{+ t\sin t} - t\sin t$$

$$\therefore h = \cos t$$

$k = y_1 + R\cos\theta$

$y_1 = \sin t - t\cos t$

$k = \sin t - t\cos t + (t)\cos t$

$k = \sin t \cancel{- t\cos t} + t\cos t$.

$k = \underline{\underline{\sin t}}$

$\therefore$ The expression ~~for~~ Centre of Curvature $\Rightarrow$

$$(\underset{h}{\underset{\downarrow}{\cos t}}, \underset{k}{\underset{\downarrow}{\sin t}})$$ Ans.