

IS/Ent603/023

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Ent 603

1) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin x$

CF: RHS = 0

$m^2 + 4m + 5 = 0$

$a = 1, b = 4, c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-4 \pm \sqrt{4}}{2} = \frac{-4 \pm 2}{2} = -2 \pm 1$

$m = -2 + 1$

$m = -2 - 1$

$a = 2, j = 1$

$y = e^{-2x} (A \cos x + B \sin x)$

$y = PI$

$f(x) = 6 \sin x$

$y = C \cos x + D \sin x$

$\frac{dy}{dx} = -C \sin x + D \cos x \quad \frac{d^2y}{dx^2} = -C \cos x - D \sin x$

Substituting

$-C \cos x - D \sin x + 4[-C \sin x + D \cos x] + 5[C \cos x + D \sin x] = 6 \sin x$

$-C \cos x - D \sin x - 4C \sin x + 4D \cos x + 5C \cos x + 5D \sin x = 6 \sin x$

$-C + 4D + 5C = 0$

$-D + 4C + 5D = 6$

$4C + 4D = 0$

$-4C + 4D = 6$

$8D = 6$

$D = 6/8 = 3/4$

substitute $D = 3/4$ in eqn 1

$-4C + 4(3/4) = 6$

$-4C + 3 = 6$

$$-4c = 3$$

$$c = 3/4$$

$$y = -3/4 \cos \theta + 3/4 \sin \theta \quad (\text{PI})$$

$$y = e^{-2\theta} (A \cos \theta + B \sin \theta) - 3/4 \cos \theta + 3/4 \sin \theta$$

$$y = e^{-2\theta} (A \cos \theta + B \sin \theta) + 3/4 (\sin \theta - \cos \theta)$$

At steady state

$$\frac{dy}{d\theta} = 0 \quad \text{and} \quad \theta = \pi$$

do

$$y = e^{-2\theta} (A \cos \theta + B \sin \theta) + 3/4 (\sin \theta - \cos \theta)$$

$$\frac{dy}{d\theta} = e^{-2\theta} (B \cos \theta - A \sin \theta) - 2e^{-2\theta} (A \cos \theta + B \sin \theta) + 3/4 (\cos \theta - \sin \theta)$$

do

$$\frac{dy}{d\theta} = e^{-2\theta} (B \cos \theta - A \sin \theta) - 2e^{-2\theta} (A \cos \theta + B \sin \theta) + 3/4 (\cos \theta - \sin \theta)$$

do

$$\frac{dy}{d\theta} = 3/4 (\sin \theta - \cos \theta)$$

do

$$\frac{dy}{d\theta} = 3/4 (\sin \theta - \cos \theta)$$

do

$$2) \quad EI \frac{d^2y}{dx^2} = \frac{w}{2} (l-x)^2$$

$$EI \frac{d^2y}{dx^2} = 0$$

$$EI m^2 = 0$$

$$m^2 = 0 \Rightarrow m = \pm \sqrt{0} = 0$$

$$m_1 = m_2 = 0$$

$$y = e^{0x} (A + Bx)$$

$$\text{SF: } y = A + Bx$$

$$y = Rx^2 + Sx^3 + Tx^4$$

$$\frac{dy}{dx} = 2Rx + 3Sx^2 + 4Tx^3$$

$$\frac{d^2y}{dx^2} = 2R + 6Sx + 12Tx^2$$

$$EI (2R + 6Sx + 12Tx^2) = w/2 (l-x)^2$$

$$2REI + 6Sx EI + 12Tx^2 EI = \frac{w}{2} [l^2 - 2lx + x^2]$$

$$4REI + 12Sx EI + 24Tx^2 EI = w l^2 - 2w l x + w x^2$$

$$24TEI = 0$$

$$T = 0$$

$$24EI$$

$$12Sx EI = 2w l$$

$$S = \frac{2w l}{24EI}$$

$$24EI$$

$$y = \left[\frac{w l^2}{4EI} \right] x^2 - \left[\frac{w l}{6EI} \right] x^3 + \left[\frac{w}{24EI} \right] x^4$$

$$y = \frac{w l^2 x^2}{4EI} - \frac{w l x^3}{6EI} + \frac{w x^4}{24EI}$$

$$y = \frac{6w l^2 x^2 - 4w l x^3 + w x^4}{24EI}$$

$$24EI$$

$$\text{P.F. } y = \frac{w}{24EI} [6l^2 x^2 - 4l x^3 + x^4]$$

$$y = A + Bx + \frac{w}{24EI} (6l^2 x^2 - 4l x^3 + x^4)$$

$$\text{at } x=0, y=0, \frac{dy}{dx} = 0$$

$$0 = A + B(0) + \frac{w}{24EI} [6l^2(0)^2 - 4l(0)^3 + (0)^4]$$

$$B = 0$$

$$\text{when } A = B = 0$$

$$y = 0 + 0x + \frac{w l}{24EI} [6l^2 x^2 - 4l x^3 + x^4]$$

$$y = \frac{w l}{24EI} [6l^2 x^2 - 4l x^3 + x^4]$$

$$\text{when } x=l$$

$$y = \frac{w l}{24EI} (6l^3 - 4l^4 + l^4) \quad , \quad y = \frac{w l}{24EI} [2l^4]$$

$$\therefore y = \frac{w l^5}{8EI}$$

