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MECH ENG
16/ENG06/046

① The parametric equation of a curve are as given as below
1 & 2

$x = \cos t + \sin t - 1$
$y = \sin t - t\cos t - 2$
in terms of determinant

② An expression for the radius of curvature

Soln

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

① $x = \cos t + t\sin t$

$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$$
$$= t\cos t$$

$y = \sin t - t\cos t$
$$\frac{dy}{dt} = \cos t - (-t\sin t + \cos t)$$
$$= \cos t + t\sin t - \cos t$$
$$= t\sin t$$
$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$= t\sin t \times \frac{1}{t\cos t} = \frac{\sin t}{\cos t}$$

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$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left[\frac{\sin t}{\cos t}\right]' \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{V\,du/dx - u\,dv/dx}{V^2}$$

where $V = \cos t$

$du = \sin t$

$u = \sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

recall that $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

Also recall that

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2}}{1/t\cos^3 t}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{1/t\cos^3 t}$$

$$R = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$= \frac{1}{(\sqrt{\cos^2 t})^3} \times t\cos^3 t$$

$$R = \frac{t\cos^3 t}{(\sqrt{\cos^2 t})^3}$$

$$R = t$$

(b) $h = x_1 - R\sin\theta - 1$

$k = y_1 + R\cos\theta - 2$

$R = t \quad t = \theta$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

$\therefore h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

$= (h, k) = (\cos t, \sin t).$