

EKANEM EFFIONG NYARAWO  
 15/ENG07/017  
 PETROLEUM ENGINEERING  
 ENG 381

$$\textcircled{1} \frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

Solution.

$$y'' + y' + 5y = 6\sin\theta$$

$$k^2 + 4k + 5 = 0$$

$$k^2 + 4k = -5$$

$$k^2 + 4k + (2)^2 = -5 + (2)^2$$

$$(k+2)^2 = -1$$

$$k+2 = \pm\sqrt{-1}$$

$$k+2 = \pm 1i$$

$$k_1 = -2 + i, \quad k_2 = -2 - i$$

$$y_h = C_1 e^{(-2+i)\theta} + C_2 e^{(-2-i)\theta}$$

$$y_h = e^{-2\theta} (A \cos\theta + B \sin\theta)$$

$$y_p = A \cos\theta + B \sin\theta$$

$$y_p' = -A \sin\theta + B \cos\theta$$

$$y_p'' = -A \cos\theta - B \sin\theta$$

$$-A \cos\theta - B \sin\theta + 4(-A \sin\theta + B \cos\theta) + 5A \cos\theta +$$

$$5B \sin\theta = 6 \sin\theta$$

$$-A \cos\theta - B \sin\theta - 4 \sin\theta + 4B \cos\theta - 6 \sin\theta$$

$$(-4A + 4B) \sin\theta + (4A + 4B) \cos\theta - 6 \sin\theta$$

$$-4A + 4B + 6$$

$$4A + 4B = 0$$

$$8B = 6, \quad B = \frac{6}{8}, \quad B = \frac{3}{4}$$

$$y_p = -\frac{3}{4} \cos\theta + \frac{3}{4} \sin\theta$$

$$y = y_h + y_p = e^{-2\theta} (A \cos\theta + B \sin\theta) + \frac{3}{4} \sin\theta = \frac{3}{4} \cos\theta$$

Steady State equations

$$y_p = 0$$

$$y_p' = \frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta = 0$$

$$\frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta = 0$$

$$\frac{3}{4} \cos \theta = -\frac{3}{4} \sin \theta$$

$$\cos \theta = -\sin \theta$$

$$\sin \theta = -\frac{\cos \theta}{\cos \theta}$$

$$\tan \theta = -1$$

$$(2) \quad EI \frac{d^2 y}{dx^2} = \frac{W}{I} (L-x)^2$$

$$EIM^2 = 0$$

$$M^2 = 0$$

$$M = \pm \sqrt{0}$$

$$M = \pm 0$$

$$y = e^{0x} [A + Bx]$$

$$y = A + Bx$$

$$y_p = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$$

$$\frac{d^2 y}{dx^2} = 2F + 6Gx + 12Hx^2$$

$$EI [2F + 6Gx + 12Hx^2] + \frac{W}{2} (L-x)^2$$

$$2FEI + 6GEIx + 12HEIx^2 = \frac{W}{2} (L-x)^2$$

$$4EI + 12EIx + 24EIx^2 = W(L - 2Lx + x^2)$$

$$24 + EI = W$$

$$H = \frac{W}{24EI} \quad \text{--- (1)}$$

$$12GEI = -2WL$$

$$G = \frac{-2WL}{12EI} = -\frac{WL}{6EI}$$



$$4FEI = WL^2$$

$$F = \frac{WL^2}{4EI}$$

$$y = \left[ \frac{WL^2}{4EI} \right] x^2 - \left[ \frac{WL}{6EI} \right] x^3 + \left[ \frac{W}{24EI} \right] x^4$$

$$= \frac{WL^2 x^2}{4EI} - \frac{WL x^3}{6EI} + \frac{W x^4}{24EI}$$

$$= \frac{6WL^2 x^2 - 4xLx^3 + Wx^4}{24EI}$$

$$G \cdot E = y = A + Bx + W \left[ \frac{6L^2 x^2 - 4Lx^3 + x^4}{24EI} \right]$$

$$a + y = 0, \quad x = 0 \quad \frac{dy}{dx} = 0$$

$$0 = A + B(0) + \frac{W}{24EI} \left[ 6L^2(0) - 4L(0) + 0 \right]$$

$$A = 0$$

$$\frac{dy}{dx} = \frac{B + W}{24EI} \left[ 12L(0) - 12(0) + 4(0) \right]$$

$$B = 0$$

$$y_p = \frac{W}{24EI} \left[ 6L^2 x^2 - 4Lx^3 + x^4 \right]$$

$$y_p = \frac{Wx^2}{24EI} \left[ 6L^2 - 4Lx + x^2 \right]$$

$$y_p = \frac{Wx^2}{24EI} \left[ x^2 - 4Lx + 6L^2 \right]$$

when  $x = L$

$$y_p = \frac{WL}{24EI} \left[ L^2 - 4L^2 + 6L^2 \right], \quad y_p = \frac{WL^2}{24EI} \left[ 3L^2 \right]$$

$$\therefore y = \frac{WL^4}{8EI}$$