

10/10/2017

Assignment

i) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$$m^2 + 4m + 5 = 0$$

$$m = -2 \pm j$$

$$\alpha = -2 \quad \beta = 1$$

$$\text{C.F. : } y = e^{-2\theta} (A \cos\theta + B \sin\theta)$$

$$\text{P.I. : } y = C \cos\theta + D \sin\theta$$

$$\frac{dy}{d\theta} = -C \sin\theta + D \cos\theta$$

$$\frac{d^2y}{d\theta^2} = -C \cos\theta - D \sin\theta$$

$$-C \cos\theta - D \sin\theta - 4C \sin\theta + 4D \cos\theta + 5C \cos\theta + 5D \sin\theta = 6 \sin\theta$$

$$\cos\theta (-C + 4D + 5C) + \sin\theta (-D - 4C + 5D) = 6 \sin\theta$$

$$4D + 4C = 0$$

$$4D - 4C = 6$$

$$C = -3/4, \quad D = 3/4$$

$$\text{P.I. : } y = -3/4 \cos\theta + 3/4 \sin\theta$$

$$\text{G.S. : } y = e^{-2\theta} (A \cos\theta + B \sin\theta) - 3/4 \cos\theta + 3/4 \sin\theta$$

ii) $\frac{dy}{d\theta} = 3/4 \sin\theta + 3/4 \cos\theta$

$$3/4 \sin\theta + 3/4 \cos\theta = 0, \quad 3/4 \sin\theta = -3/4 \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}, \quad \tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta \text{ at steady state} = -45^\circ$$

$$\theta = -45^\circ$$

$$2 \quad EI \frac{d^2 y}{dx^2} = \frac{w}{2} (L-x)^2$$

Solution

C.F

$$EI \cdot m^2 = 0$$

$$m^2 = 0$$

$$m = \sqrt{0}$$

$$m = 0$$

$$y = e^{0x} (A+Bx)$$

$$y = A+Bx$$

P.I

$$y = Gx^2 + Hx^3 + 2Zx^4$$

$$\frac{dy}{dx} = 2Gx + 3Hx^2 + 4Zx^3$$

$$\frac{d^2 y}{dx^2} = 2G + 6Hx + 12Zx^2$$

$$EI (2G + 6Hx + 12Zx^2) = \frac{w}{2} (L-x)^2$$

$$2GEI + 6HxEI + 12Zx^2EI = \frac{w}{2} (L^2 - 2Lx + x^2)$$

$$4GEI + 12HxEI + 24Zx^2EI = w(L^2 - 2Lx + x^2)$$

Comparing Coefficients

$$24ZEI = w$$

$$Z = \frac{w}{24EI}$$

$$12HEI = -2wL$$

$$H = \frac{-2wL}{12EI}$$

$$H = \frac{-wL}{6EI}$$

$$4GGE = WL^2$$

$$G = \frac{WL^2}{4GE}$$

$$y = \left(\frac{WL^2}{4EI} \right) x^2 + \left(\frac{-WL}{6EI} \right) x^3 + \left(\frac{W}{24EI} \right) x^4$$

$$y = \frac{WL^2 x^2}{4EI} - \frac{WLx^3}{6EI} + \frac{Wx^4}{24EI}$$

$$y = \frac{6WL^2 x^2 - 4WLx^3 + Wx^4}{24EI}$$

$$y = \frac{W}{24EI} (6L^2 x^2 - 4Lx^3 + x^4)$$

G.S

$$y = A + Bx + \frac{W}{24EI} (6L^2 x^2 - 4Lx^3 + x^4)$$

$$\text{at } y=0 \quad x=0 \quad \frac{dy}{dx} = 0$$

$$0 = A \quad A = 0$$

$$\frac{dy}{dx} = 0 + \frac{W}{24EI} (12L^2 x - 12Lx^2 + 4x^3)$$

$$0 = B + \frac{W}{24EI} (12L^3(0) - 12L(0)^2 + 4(0)^3)$$

$$B = 0$$

P.S

$$y = \frac{W}{24EI} (6L^2 x^2 - 4Lx^3 + x^4)$$

$$y = \frac{W}{24EI} x^2 (6L^2 - 4Lx + x^2) \quad x=L$$

$$y = \frac{WL^2}{24EI} (6L^2 - 4L^2 + L^2)$$

$$y = \frac{WL^2}{24EI} (3L^2)$$

$$y = \frac{3WL^4}{24EI} = \frac{y = WL^4}{8EI}$$

