15/ENG06/017
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Question 1

$$\frac{d^2y}{d\theta} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta \quad \text{——— (1)}$$

$M^2 + 4M + 5 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$\frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$\frac{-4 \pm \sqrt{-1}\sqrt{4}}{2} = \frac{-4 \pm 2j}{2}$$

$-2 \pm j$

$M_1 = -2 + j \qquad M_2 = -2 - j$

CF $\therefore y = e^{-2\theta}(C\cos\theta + D\sin\theta)$

PI $\therefore y = C\cos\theta + D\sin\theta$

$$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta \quad (II)$$

$$\frac{d^2y}{d\theta} = -C\cos\theta - D\sin\theta \quad (III)$$

~~Substitute $\frac{d^2y}{d\theta}$ into~~

Substitute (III) and (II) into (1)

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) +$
$5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta$
$= 6\sin\theta$

$-C\cos\theta + 4D\cos\theta + 5C\cos\theta - D\sin\theta - 4C\sin\theta +$
$5D\sin\theta = 6\sin\theta$

$\cos\theta(-C + 4D + 5C) + \sin\theta(-D - 4C + 5D)$
$= 6\sin\theta$

$\cos\theta(4D + 4C) + \sin\theta(-4C + 4D) = 6\sin\theta$

Comparing Coefficient

$4D + 4C = 0$ ——— (1)

$-4C + 4D = 6$ ——— (2)

$4C + 4D = 0$

$\ominus$ $-4C + 4D = 6$

$4C - -4C = -6$

$8C = -6$

$C = \frac{-6}{8}$

$C = \frac{-3}{4}$

sub $\frac{-3}{4}$ for C in equation (1)

$4D + 4\left(\frac{-3}{4}\right) = 0$

$4D + -3 = 0$

$4D - 3 = 0$

$4D = 3$

$D = \frac{3}{4}$

$y = C\cos\theta + D\sin\theta$

$\therefore y = \frac{-3}{4}\cos\theta + \frac{3}{4}\sin\theta$

$GS = e^{-2\theta}\left(\frac{-3}{4}\cos\theta + \frac{3}{4}\sin\theta\right) + \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

at $\theta = \infty$

and $\frac{dy}{d\theta} = 0$

$\frac{dy}{d\theta} = e^{-2\theta}(-C\sin\theta + D\cos\theta) + (C\cos\theta + D\sin\theta) - 2e^{-2\theta} +$
$\frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

at $\theta = \infty$ and $\frac{dy}{d\theta} = 0$

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$-\cancel{\frac{3}{4}}\cos\theta = \cancel{\frac{3}{4}}\sin\theta$

$-\cos\theta = \sin\theta$

divide both sides by $-\cos\theta$

$\cancel{\frac{-\cos\theta}{-\cos\theta}} = \frac{\sin\theta}{-\cos\theta}$

$1 = \frac{-\sin\theta}{\cos\theta}$

$1 = -\tan\theta$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45°$

Question 2

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

$EF\ m^2 = 0$

$m^2 = 0$

$m = \pm 0$

$y = e^{ax}(A+Bx)$

$y = A+Bx$ ———— CF

to obtain Particular integral

$y = Px^2 + Qx^3 + Rx^4$ ———— (I)

$\frac{\Delta y}{\Delta x} = 2Px + 3Qx^2 + 4Rx^3$

$\frac{\Delta^2 y}{\Delta x^2} = 2P + 6Qx + 12Rx^2$

$EI\left[2P + 6Qx + 12Rx^3\right] = \frac{w}{2}(L-x)^2$

$2PEI + 6QEIx + 12REIx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4PEI + 12QEIx + 24REIx^2 = w(L^2 - 2Lx + x^2)$

$24REI = w$

$R = \frac{w}{24EI}$ ———— (II)

$12QEI = -2wL$

$Q = \frac{-\cancel{2}wL}{\cancel{12}EI}$

$Q = \frac{-wL}{6EI}$ ———— (III)

$4PEI = wL^2$

$P = \frac{wL^2}{4EI}$ ———— (IV)

Substitute (II), (III) and (IV) into (I)

$y = Px^2 + Qx^3 + Rx^4$

$y = \left(\frac{wL^2}{4EI}\right)x^2 + \left(\frac{-wL}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)x^4$

$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$

$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$

$y = \frac{w}{24EI}(L^2x^2 - Lx^3 + x^4)$ ———— PI

G.S

$$y = A + Bx + \frac{w}{24EI}\left(6l^2x^2 - 4lx^3 + x^4\right)$$

at $y = 0$, $x = 0$ $\quad \frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left(6l^2(0) - 4l(0) + 0\right)$$

$$0 = A + 0 + 0$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left(12l^2x - 12lx^2 + 4x^3\right)$$

$$0 = B + \frac{w}{24EI}\left(12l^2(0) - 12(0)x^2 + 4(0)^3\right)$$

$$0 = B + 0$$

$$B = 0$$

Particular ~~Sol~~ Solution

$$y = \frac{w}{24EI}\left(6l^2x^2 - 4lx^3 + x^4\right)$$

$$y = \frac{wx^2}{24EI}\left(6l^2 - 4lx + x^2\right)$$

$$y = \frac{wx^2}{24EI}\left(x^2 - 4lx + 6l^2\right)$$

When $x = l$

$$y = \frac{wl^2}{24EI}\left(l^2 - 4l^2 + 6l^2\right)$$

$$y = \frac{wl^2}{24EI}\left(3l^2\right)$$

$$y = \frac{wl^4}{8EI}$$