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$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta \quad \text{— i}$$

$$m^2 + 4m + 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$\frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$\frac{-4 \pm \sqrt{-1}\sqrt{4}}{2} = \frac{-4 \pm 2i}{2}$$

$$-2 \pm i$$

$$m_1 = -2 + i \qquad m_2 = -2 - i$$

$$CF: y = e^{-2\theta}(C\cos\theta + D\sin\theta)$$

$$PI: y = C\cos\theta + D\sin\theta$$

$$dy/d\theta = -C\sin\theta + D\cos\theta \quad \text{— ii}$$

$$d^2y/d\theta^2 = -C\cos\theta - D\sin\theta \quad \text{— iii}$$

Sub (iii) & (ii) in (i)

$$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$$

$$-C\cos\theta + 4D\cos\theta + 5C\cos\theta - D\sin\theta - 4C\sin\theta + 5D\sin\theta = 6\sin\theta$$

$$\cos\theta(-C + 4D + 5C) + \sin\theta(-D - 4C + 5D) = 6\sin\theta$$

$$\cos\theta(4D + 4C) + \sin\theta(-4C + 4D) = 6\sin\theta$$

Comparing coeffs

$$4D + 4C = 0 \quad \text{— iv}$$

$$-4C + 4D = 6 \quad \text{— v}$$

$$4C + 4D = 0 \Rightarrow 4D = -4C$$

Sub in iv

$$4C - -4C = -6$$

$$8C = -6$$

$$C = -3/4$$

$$\therefore 4D + 4(-3/4) = 0$$

$4D + -3 = 0$

$4D = 3$

$D = 3/4$

$y = C\cos\theta + D\sin\theta$

$\therefore y = -3/4\cos\theta + 3/4\sin\theta$

$G_S = e^{-2\theta}\left(-3/4\cos\theta + 3/4\sin\theta\right) + 3/4\cos\theta + 3/4\sin\theta$

at $\theta = \infty$

and $dy/d\theta = 0$

$dy/d\theta = e^{-2\theta}\left(-C\sin\theta + D\cos\theta\right) + (C\cos\theta + D\sin\theta) - 2e^{-2\theta} + 3/4\sin\theta + 3/4\cos\theta$

at $\theta = \infty$ & $dy = 0$

dividing both sides by $-\cos\theta$

$$\cancel{\frac{-\cos\theta}{-\cos\theta}} = \frac{\sin\theta}{-\cos\theta}$$

$1 = \frac{-\sin\theta}{\cos\theta} \Rightarrow 1 = -\tan\theta$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\cancel{\theta = 45^0}$

$\theta = -45^0$

2. $EI\, d^2y/dx^2 = \frac{w}{2}(L-x)^2$

$EIm^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$y = e^{0x}(A + Bx)$

$y = A + Bx$ ——— CF

3 Obtain particular integral

$y = Px^2 + Qx^3 + Rx^4$

$dy/dx = 2Px + 3Qx^2 + 4Rx^3$

$d^2y/dx^2 = 2P + 6Qx + 12Rx^2$

$EI(2P + 6Q + 12Rx^3] = w/2 (L-x)^2$

$2PEI + 6QEIx + 12REIx^2 = w/2 (L^2 - 2Lx + x^2)$

$4PEI + 12QEIx + 24REIx^2 = w(L^2 - 2Lx + x^2)$

$24REI = w$

$R = w/24EI$ ——— (II)

$12QEI = -2wL$

$Q = \frac{-2wL}{12EI} \Rightarrow \frac{-wL}{6EI}$ ——— (III)

$4PEI = wL^2$

$P = \frac{wL^2}{4EI}$ ——— (IV)

Sub (II)(III) & (IV) in (I)

$y = Px^2 + Qx^3 + Rx^4$

$y = (wL^2/4EI)x^2 + (-wL/6EI)x^3 + \left(\frac{w}{24EI}\right)x^4$

$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$

$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$

$y = \frac{w}{24EI}(L^2x^2 - Lx^3 + x^4)$ ———→ PI

Q.5

$y = A + Bx + \frac{w}{24EI}(6L^2x^2 - 4Lx^3 + x^4)$

at $y = 0$, $x = 0$ $dy/dx = 0$

$0 = A + B(0) + w/24EI(6L^2(0) - 4L(0) + 0)$

$0 = A + 0 + 0$

$A = 0$

$dy/dx = B + \frac{w}{24EI}(12L^2x - 12Lx^2 + 4x^3)$

$0 = B + \frac{w}{24EI}(12(L^2 0) - 12(0)x^2 + 4(0)^3)$

$0 = B + 0$

$B = 0$

Particular solution

$y = w/24EI\,(6L^2x^2 - 4Lx^3 + x^4)$

$y = \dfrac{wx^2}{24EI}(6L^2 - 4Lx + x^2)$

$y = wx^2/24EI\,(x^2 - 4Lx + 6L^2)$

When $x = L$

$y = wL^2/24EI\,(L^2 - 4L^2 + 6L^2)$

$y = wL^2/24EI\,(3L^2)$

$y = wL^4/8EI$

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