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ENG 281

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1. The parametric equations of curve are as given in equations ① and ②.

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

In terms of t, determine ~~an~~

i) ~~$x = \cos t + t\sin t$~~ an expression for the radius of curvature

ii) an expression for the coordinates (h, k) of the centre of curvature.

Solution

1: $x = \cos t + t\sin t$

~~$\frac{dx}{dt} = -\sin t + (\cos t + t$~~

$\frac{dx}{dt} = -\sin t + (t\cos t + \sin t)$ — Using product rule

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t - (-t\sin t + \cos t)$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t$

$\frac{dy}{dt} = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$

$dy/dx = \frac{\sin t}{\cos t}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx}\frac{dt}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\frac{dy}{dx}\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{\sin t}{\cos t}\right]\frac{dt}{dx}$$

$$= \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2}\frac{dt}{dx}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \cdot \frac{1}{t\cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{t\cos^3 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$= \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$= \left[1^{3/2} + \left(\frac{\sin t}{\cos t}\right)^{2 \times \frac{3}{2}}\right]^{3/2} \times t\cos^3 t$$

$$= \left[1 + \left(\frac{\sin t}{\cos t}\right)^3\right] \times t\cos^3 t$$

$$= \frac{\cos^3 t + \sin^3 t}{\cos^3 t} \times t\cos^3 t$$

$$= \cos^3 t + $$

$$R = \left[1 + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{1}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \frac{1}{\cos^3 t} \times t\cos^3 t$$

$$R = t$$

iii $h = x_1 - \rho \sin\theta$

$k = y_1 + \rho \cos\theta$

$\tan\theta = \left[\frac{dy}{dx}\right]$

$\tan\theta = \frac{\sin t}{\cos t}$

$\theta = \tan^{-1}\left(\frac{\sin t}{\cos t}\right)$

$\theta = \tan^{-1}(\tan t)$

$\theta = t$

$h = \cos t + t\sin t - t\sin t$

$h = \underline{\cos t}$

$k = \sin t - t\cos t + t\cos t$

$k = \underline{\sin t}$

$\therefore$ The center of curvature $C(h, k) = (\cos t, \sin t)$