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15/ENG05/008
MECHATRONICS
ENG 381

1) $d^2y/d\theta^2 + 4\,dy/d\theta + 5y = 6\sin\theta$

Assume homogeneity

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 0$$

$$k^2 + 4k + 5 = 0$$

$a = 1,\ b = 4,\ c = 5$ using $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$k = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} \Rightarrow \frac{-4 \pm \sqrt{-4}}{2}$$

$$k = \frac{-4 \pm 2j}{2} \quad \therefore\ k = -2 \pm j$$

CF $y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

$$y = (C\cos\theta + D\sin\theta)$$

$$dy/d\theta = -C\sin\theta + D\cos\theta$$

$$-[C\cos\theta - D\sin\theta] + 4[-C\sin\theta + D\cos\theta] + 5[C\cos\theta + D\sin\theta] = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$$

$$-C + 4D + 5C = 0$$

$$-D + 4C + 5D = 6$$

$$4C + 4D = 0 \quad \text{——— ①}$$

$$+\ -4C + 4D = 6 \quad \text{——— ②}$$

$$8D = 6$$

$$D = 6/8 = 3/4$$

Substitute $D = 3/4$ in equation ②

$$-4C + 4(3/4) = 6$$

$$-4C + 3 = 6$$

$$-4C = 3 \quad \therefore\ C = -3/4$$

$y = -3/4 \cos\theta + 3/4 \sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4 \cos\theta + 3/4 \sin\theta$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\sin\theta - \cos\theta)$

at steady state

$dy/dx = 0$ and $\theta = \alpha$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\sin\theta - \cos\theta)$

$dy/d\theta = e^{-2\theta}(B\cos\theta - A\sin\theta) - 2e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4(\sin\theta - \cos\theta)$

$dy/d\theta = e^{-2\alpha}(B\cos\alpha - A\sin\alpha) - 2e^{-2\alpha}(A\cos\alpha + B\sin\alpha) + 3/4(\sin\alpha - \cos\alpha)$

$dy/d\theta = 3/4(\sin\alpha - \cos\omega)$

$dy/d\theta = 3/4(\sin\theta - \cos\theta)$

2) $EI\, d^2y/dx^2 = w/2 (L-x)^2$

$$\frac{d^2y}{dx^2} = \frac{w}{2EI}(L-x)^2$$

$$y'' = \frac{w(L-x)^2}{2EI}$$

$$k^2 = \frac{w(L-x)^2}{2EI} \quad \therefore \quad k = \pm\sqrt{\frac{w(L-x)^2}{2EI}}$$

$$y = C_1 \cosh\left(\sqrt{\frac{w(L-x)^2}{2EI}}\right) + C_2 \sinh\left(\sqrt{\frac{w(L-x)^2}{2EI}}\right)$$

$y = Rx^2 + Sx^3 + Tx^4$

$dy/dx = 2Rx + 3Sx^2 + 4Tx^3$

$d^2y/dx^2 = 2R + 6Sx + 12Tx^2$

$EI[2R + 6Sx + 12Tx^2] = w/2 (L-x)^2$

$2REI + 6SxEI + 12Tx^2EI = w/2 [L^2 - 2Lx + x^2]$

$4REI + 12SxEI + 24Tx^2EI = wL^2 - 2wLx + wx^2$

$24TEI = w$

$$T = \frac{w}{24EI}$$

$$12\,S\,EI = -2wL$$

$$S = \frac{-2wL}{24EI}$$

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y = \frac{6wL^2w^2 - 4wLx^3 + wx^4}{24EI}$$

$$PI = y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = C_1 \operatorname{Cosh}\left(\sqrt{\frac{w(L-x)^2}{2EI}}\right) + C_2 \operatorname{Sinh}\left[\left(\sqrt{\frac{w(L+x)^2}{2EI}}\right) + \frac{w}{24EL}\left[6L^2x^2 - 4Lx^2 + \right.\right.$$

at $x=0$, $y=0$, $dy/dx = 0$

$$0 = C_1 \operatorname{Cosh}\left(\sqrt{\frac{w(L-0)^2}{2EI}}\right) + C_2 \operatorname{Sinh}\left(\sqrt{\frac{w(L-0)^2}{2EI}}\right) + \frac{w}{24EI}\left[6L^2(0)^2 - 4L(0)^2 - (0)\right.$$

$$0 = C_1 \operatorname{Cosh}\left(\sqrt{\frac{wL^2}{2EI}}\right) + C_2 \operatorname{Sinh}\left(\sqrt{\frac{wL^2}{2EI}}\right) + 0$$

~~Substitute through log~~

when $x=L$

$$y = \frac{w}{24EL}\left[6L^4 - 4L^4 + L^4\right]$$

$$y = \frac{w}{24EL}\left[3L^4\right]$$

$$y = \frac{wL^4}{8EI}$$

graph1 Sheet1