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 Mechanical Eng
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 207-207

$$2) \quad x = \cos t + t \sin t \\ y = 2 \sin t - t \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{dy}{dt} = -\sin t + (t \cos t + \sin t)$$

$$\frac{dy}{dt} = -\sin t + t \cos t + \sin t$$

$$\frac{dx}{dt} = t \cos t$$

$$\frac{dy}{dx} = \cos t \cdot (t \sin t + \cos t)$$

$$\frac{dy}{dx} = \cos t - t \sin t + \cos t$$

$$\frac{dy}{dx} = 2 \cos t - t \sin t$$

$$\frac{dy}{dx} = \frac{2 \cos t - t \sin t}{t \cos t} = \frac{2 \cos t}{t \cos t} - \frac{t \sin t}{t \cos t}$$

$$\frac{dy}{dx} = \frac{2}{t} - \frac{\sin t}{\cos t} = \frac{2}{t} - \tan t$$

$$2t^{-1} - \tan t$$

$$\frac{d^2y}{dx^2} = -2t^{-2} - \sec^2 t$$

$$\frac{d^2y}{dx^2} = d \left(\frac{dy}{dx} \right) \times \frac{dt}{dx} = \frac{d^2y}{dt^2}$$

$$\frac{dy}{dx} = \frac{2t^{-2} - 6t^2}{t \cos t}$$

Since $\sec t = \frac{1}{\cos t}$; $\sec^2 t = \left(\frac{1}{\cos t}\right)^2$

$$\frac{d^2y}{dx^2} = \frac{-2t^{-3} - \left(\frac{1}{\cos t}\right)^2}{t \cos t}$$

$$\frac{d^2y}{dx^2} = \frac{-2t^{-3}}{t \cos t} - \frac{1}{\cos^3 t}$$

$$\frac{d^2y}{dy^2} = 2t^{-2} \sec^2 t - \frac{1}{\cos^3 t}$$

$$\frac{d^2y}{dy^2} = \left(\frac{-2t^{-3}}{\cos t} - \frac{1}{\cos^3 t} \right) = \frac{2t^{-3}}{\cos t} + \frac{1}{\cos^3 t}$$

$$R = \left(1 + \left(\frac{dy}{dx} \right)^2 \right)^{1/2}$$

$$R = \left[1 + \left(\frac{2t^{-3}}{\cos t} + \frac{1}{\cos^3 t} \right)^2 \right]^{1/2}$$

ii $x = R \sin \theta = H$

$y = R \cos \theta = F$

$\cos t + \tan t - R \sin \theta = H$

$\sin t + \tan t - R \cos \theta = K$

$$H = \cos t + \tan t - \left[\frac{(1 + 2t^{-3} \tan t + t^{-6})^{1/2}}{2t^{-3} + \frac{1}{\cos t}} \right] \sin(\tan^{-1} \left(\frac{\sin t + \tan t}{\cos t + \tan t} \right))$$

$dx = -\sin t$
 dt

1) Radius of Curvature (R)

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$R = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2} \times \frac{dx^2}{d^2y}$$

$$R = \left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2} \times t \cos^3 t$$

$$R = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times t \cos^3 t$$

$$R = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times t \cos^3 t$$

From trig we know $\sin^2 x + \cos^2 x = 1$

$$R = \sqrt{\left(\frac{1}{\cos^2 t}\right)^3} \times t \cos^3 t$$

$$R = \frac{1}{\sqrt{(\cos^2 t)^3}} \times t \cos^3 t$$

$$R = \frac{1}{\cos^2 t \times \sqrt{\cos^2 t}} \times t \cos^3 t$$

$$R = \frac{1}{\cos^2 t} \times t \cos^3 t$$

$$R = t$$

$\frac{\sin t + \cos t}{\cos t + \sin t}$

Coordinate of centre of Curvature (h, k)

$$h = x_1 - r \sin \theta$$

$$k = y_1 + r \cos \theta$$

$$x_1 = \cos t + t \sin t$$

$$r = t$$

$$y_1 = \sin t - t \cos t$$

$$\theta = t$$

$$h = \cos t + t \sin t - t \sin t$$

$$h = \cos t$$

$$k = \sin t - t \cos t + t \cos t$$

$$k = \sin t$$

$$\therefore (h, k) = (\cos t, \sin t)$$