OYEWALE SEMILORE
MECHANICAL ENGINEERING
16/ENG 06/065
ENG 281 ASSIGNMENT

The parametric equations of a curve are as given on equations (1) and (2)

$$x = \cos t + t \sin t$$
$$y = \sin t - t \cos t$$

In terms of t determine:
i) an expression for the radius of curvature (R) and
(ii) expressions for the coordinates (h, k) of the centre of curvature

solution

$x = \cos t + t \sin t$
$y = \sin t - t \cos t$

$$\frac{dx}{dt} = -\cancel{\sin t} + \cancel{\sin t} + t\cos t$$

$$\frac{dx}{dt} = t\cos t$$

$$\frac{dy}{dt} = \cos t - \cos t + t\sin t$$

$$\frac{dy}{dt} = t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

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$$\frac{dy}{dx} = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\left(\frac{dt}{dx}\right)$$

Applying Quotient rule

$u = \sin t \qquad v = \cos t$

$$\frac{d^2y}{dx} = \frac{v\, du/dx - u\frac{dv}{dx}}{v^2}$$

$= \cancel{\cos t}$

$$\frac{du}{dt} = \cos t$$

$$\frac{dy}{dt} = -\sin t$$

$$\therefore \frac{d^2y}{dx} = \frac{\cos t \cdot \cos t - (\sin t)(-\sin t)}{\cos^2 t} \times \frac{dt}{dx}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

From trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\therefore \frac{d^2y}{dx} = \frac{1}{t\cos^3 t}$$

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$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \left[1+\left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \left[1+\frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$R = \left[\frac{\cos^2 t+\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$R = \frac{1}{\cancel{\cos^3 t}} \times t\,\cancel{\cos^3 t}$$

$R = t.$

∴ ~~Radius of curvature in t~~

∴ The expression of Radius of curvature (R) is t

Recall

$h = x_1 - R\sin\theta$

$k = y_1 + R\cos\theta$

$R = t \quad , \quad \theta = t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

$\therefore \quad h = \cos t + t\sin t - (t\sin t)$

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$\therefore h = \cos t$.

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

The expressions for the coordinates of (h, k) of the centre of curvature is $(\cos t, \sin t)$.

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