ASSIGNMENT 1

EDET, UBONG UMOH

16/ENG06/025

MECHANICAL ENGR.

ENG281

17/10/2017.

1 (a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$

$= \left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \right) \sin(\cos \pi/2)$

$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \sin\left(\cos \frac{\pi}{2}\right)$

Thus undeterminate ∴ with L'hopital, dy/dx of the numerator $= U\frac{dv}{dx} + V\frac{du}{dx}$

$\frac{dy}{dx} = \begin{cases} \text{let } u = x^2 - \pi/4 \\ V = \sin(\cos x) \end{cases}$

$\frac{du}{dx} = 2x$ and $\frac{dy}{dx} = \sin(\cos x)$ = let $\cos x = w$

$V = \sin w$

$\frac{dv}{dw} = \cos w$

$\frac{dw}{dx} = -\sin x$

$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx} = -\sin x \cos(\cos x).$

$\Rightarrow \frac{\left(x^2 - \frac{\pi}{4}\right) x - \sin \cos(\cos x) + \sin(\cos x)(2x)}{1}$

$\Rightarrow \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \times -\sin 90 \cos(\cos 90) + \sin \cos 90 \times 2(\pi/2)$

$\Rightarrow \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \times -1 * 0 \times \pi$

$= -\frac{\pi^2}{4} + \frac{\pi}{4}$; $= \pi/4 - \frac{\pi^2}{4}$

$$\lim_{x=\pi/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{x-\pi/2}\right] \to \frac{\pi(1-\pi)}{4}$$

(1b) $\lim_{x\to\pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$

$\Rightarrow \lim \ln\left[\exp\left[\frac{(3x-1)(x+1)}{x+1^2}\right]\right]$

$\Rightarrow \lim_{x\to\pi/2} \ln(\exp(3x-1)]$

$\Rightarrow \lim_{x\to\pi/2}(3x-1) = 3(\pi/2)-1$

$\Rightarrow \frac{3x}{2} - 1 = \frac{3\pi}{2} - \frac{1}{1}$

$\lim \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right] = \frac{3\pi-2}{2}$

(1c) $\lim_{x\to 2+\sqrt{3}} \cos\left(\frac{\sin^{-1}(x-2)}{x-\sqrt{3}}\right)$

$\lim_{x\to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3})}\right)$

$\cos\left(\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]\right)$

$\Rightarrow \cos(\sin^{-1}(0.8660)$

$\Rightarrow \cos 60^\circ$

$\Rightarrow 1/2$ // Ans.

1d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$

$\lim_{x \to 4} \left[\frac{(x-4)(x-4)}{(x-4)(x-1)}\right]$

$\lim_{x \to 4} \left[\frac{x-4}{x-1}\right] \Rightarrow \frac{4-4}{4-1} = \frac{0}{3} = 0$ Ans

2a) $U_n = \frac{2}{(n+1)(n+2)}$

$U_{n+1} = \frac{2}{(n+2)(n+3)}$

ratio $\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$\frac{U_{n+1}}{U_n} = \frac{n+1}{n+3}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \lim_{n \to \infty} \frac{n+1}{n+3}$

$\Rightarrow \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}} = \frac{1 + \frac{1}{n}}{1 + \frac{3}{n}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$

Since $\lim_{x \to \infty} \frac{U_{n+1}}{U_n}$

$\therefore$ The series is conclusive

2b) $\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\frac{1}{n^2}\right]=\sum_{n=1}^{\infty}\frac{1}{n^p}$

$\rightarrow\left[\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\cdots\frac{2}{n^2}\right]=\sum_{n=1}^{\infty}=\frac{2}{n^2}$

$\therefore p=2$

Thus $p>1$, Series will converge

3c) $U_n=\frac{x^n}{(2n+1)^3}$, $U_{n+1}=\frac{x^{n+1}}{(2n+2)^3}$

$\lim\frac{U_{n+1}}{U_n}=\frac{x^{n+1}}{(2n+2)^3}\times\frac{(2n+2)^3}{x^n}$

$\frac{x(2n+1)^3}{(2n+2)^3}=\frac{8n^3+12n^2+n+1}{8n^3+24n^2+24n+8}$

Thus $\frac{\left(8+\frac{2}{n}+\frac{1}{n^2}+\frac{1}{n^3}\right)}{\left(8+\frac{12}{n}+\frac{24}{n^2}+\frac{8}{n^3}\right)}$ as $n\rightarrow\infty$

$\frac{1}{n}=0$

$\frac{8x}{8}\geq x-1$

$\therefore x<1$ Ans.

4) $\lim_{x\to 0}\left[\frac{\sin x-\cos x}{x^3}\right]$

Using L'hospital Rule

$y=\left(\frac{\sin x-\cos x}{x^3}\right)$

$\frac{dy}{dx}=\left[\frac{\cos x+\sin x}{3x^2}\right]$

$\frac{d^2y}{dx^2}=\frac{-\sin x+\cos x}{6x}$

$\frac{d^3y}{dx^3}=\frac{-\cos x-\sin x}{6}$

$\lim_{x\to 0}=\frac{-\cos 0-\cos 0}{6}\ \frac{-1-0}{6}=-\frac{1}{6}$

$\Rightarrow\lim_{x\to 0}\left[\frac{\sin x-\cos x}{x^3}\right]=-\frac{1}{6}$ A