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15/ENG07/018
Petroleum Engineering

~~$Y_p = Ax - B = -2x+7$~~
~~$Y = C_1 e^{-1/3x} + C_2 e^{x} - 2x + 7$~~

i) $dy - \frac{dy}{dx} - 2y = 8$

Solution

Assume homogenity

$Y'' - Y' - 2y = 0$

$Y = e^{kx}$

$Y' = ke^{kx}$

$Y'' = k^2 e^{kx}$

$\therefore k^2 e^{kx} - ke^{kx} - 2e^{kx} = 0$

$[k^2 - k - 2]e^{kx} = 0$

$k^2 - k - 2 = 0$

$k^2 - 2k + k - 2 = 0$

$(k^2 - 2k) + (k - 2) = 0$

$k(k-2) + 1(k-2) = 0$

$(k+1)(k-2) = 0$

$k + 1 = 0$

$k - 2 = 0$

$k_1 = -1$ & $k_2 = 2$

$Y_1 = e^{k_1 x} = e^{-x}$

$Y_2 = e^{k_2 x} = e^{2x}$

$Y = C_1 Y_1 + C_2 Y_2$

$Y = C_1 \times e^{-x} + C_2 \times e^{2x}$

$Y = C_1 e^{-x} + C_2 e^{2x}$

$Y' = 0$

$Y'' = 0$

$Y_p = c$

Substitute into $Y'' - Y' - 2y = 8$

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4$

$\therefore Y = C_1 e^{x} + C_2 e^{2x} + c$

Sub

$Y = C_1 e^{-x} + C_2 e^{2x} - 4$

② $\frac{d^2y}{dx^2} - 4Y = 10e^{3x}$

Solution

$Y'' - 4y = 0$

$Y = e^{kx}$

$Y' = ke^{kx}$

$Y'' = k^2 e^{kx}$

$k^2 e^{kx} - 4e^{kx} = 0$

$e^{kx}(k^2 - 4) = 0$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \sqrt{4}$

$\therefore k_1 = +2$ & $k_2 = -2$

$Y_1 = e^{k_1 x} = e^{2x}$

$Y_2 = e^{k_2 x} = e^{-2x}$

$Y = C_1 Y_1 + C_2 Y_2$

$Y = C_1 \times e^{2x} + C_2 \times e^{-2x}$

$Y = C_1 e^{2x} + C_2 e^{-2x}$

$Y_p = Ae^{3x}$

$Y' = 3Ae^{3x}$

$Y'' - 9Ae^{3x}$

Substitute into $Y'' - Y' - 4Y = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$e^{3x}[9A - 4A] = 10e^{3x}$

$\frac{5A}{5} = \frac{10}{5}$

$A = 2$

$Y = C_1 e^{2x} + C_2 e^{-2x} + Ae^{3x}$

$Y = C_1 e^{2x} + C_2 e^{-2x} + 2e^{3x}$

3] $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

Solution

$y'' + 2y' + y = 0$

when $y = e^{kx}$, $y' = ke^{kx}$

$y'' = k^2e^{kx}$

$\therefore k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$[k^2 + 2k + 1]e^{kx} = 0$

$k^2 + 2k + 1 = 0$

$(k^2 + k) + (k + 1) = 0$

$k(k+1) + 1(k+1) = 0$

$k + 1 = 0$

$\therefore k_1 = -1$ & $k_2 = -1$

$y_1 = e^{k_1x} = e^{-x}$

$y_2 = e^{k_2x} = e^{-x}$

$y = C_1y_1 + C_2y_2$

$y = C_1e^{-x} + C_2e^{-x}$

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

Substitute into equation

$4Ae^{-2x} + 2[-2Ae^{-2x}] + Ae^{-2x} = e^{-2x}$

$\cancel{4Ae^{-2x}} - \cancel{4Ae^{-2x}} + A\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$\therefore A = 1$

$y = C_1e^{-x} + C_2e^{-x} + e^{-2x}$

$y = e^{-x}(C_1 + C_2) + e^{-2x}$

4] $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

Solution

$y'' + 25y = 0$

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} + 25e^{kx} = 0$

$e^{kx}(k^2 + 25) = 0$

$k^2 + 25 = 0$

$k^2 = -25$

$k = \pm\sqrt{-25}$

$k_1 = +5i$ & $k_2 = -5i$

$y_1 = e^{k_1x} = e^{5ix}$

$y_2 = e^{k_2x} = e^{-5ix}$

$y = C_1y_1 + C_2y_2$

$y = C_1e^{5ix} + C_2e^{-5ix}$

$y_p = Ax^2 + Bx + C$

$\therefore y' = 2Ax + B$

$y'' = 2A$

Substitute into $y'' + 25y = 5x^2 + x$

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25A\cancel{x^2} = 5\cancel{x^2}$

$25A = 5$

$A = 1/5$

$25B\cancel{x} = \cancel{x}$

$25B = 1$

$B = 1/25$

$2A + 25C = 0$

$2(1/5) + 25C = 0$

$25C = -2/5$

$C = -2/125$

$\therefore y_p = Ax^2 + Bx + C = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = C_1e^{5xi} + C_2e^{-5xi} + \frac{1}{125}(25x^2 + 5x - 2)$

$y = C_1\cos 5t + C_2\sin 5t + \frac{1}{125}(25x^2 + 5x - 2)$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Solution

$y'' - 2y' + y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} - 2ke^{kx} + e^{kx} = 0$

$[k^2 - 2k + 1]e^{kx} = 0$

$k^2 - 2k + 1 = 0$

$(k^2 - k) - (k + 1) = 0$

$k((k-1) - 1(k-1) = 0$

$k - 1 = 0$

$\therefore k_1 = 1$ & $k_2 = 1$

$y_1 = e^{k_1 x} = e^x$

$y_2 = e^{k_2 x} = e^x$

$y = c_1y_1 + c_2y_2$

$= c_1e^x + c_2e^x$

$y_p = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

Substitute

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + b\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x = 4\sin x$

$(\cancel{-A} + 2B + \cancel{A})\sin x + (\cancel{-B} - 2A + \cancel{B})\cos x = 4\sin x$

$\cancel{2B\sin x = 2A\cos x}$

$2B\sin x = 4\sin x$

$B = 2$

$-2A\cos x = 0\cos x$

$A = 0$

$y = c_1e^x + c_2e^x + y_p$

$y = c_1e^x + c_2e^x + A\sin x + B\cos x$

$y = c_1e^x + c_2e^x + 0\sin x + 2\cos x$

$y = e^x(c_1 + c_2) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Solution

$y'' + 4y' + 5y = 0$

$y = e^{kx}$

$y = ke^{kx}$

$y = k^2e^{kx}$

$k^2e^{kx} + 4ke^{x} + 5e^{x} = 0$

$[k^2 + 4k + 5]e^{x} = 0$

$k^2 + 4k + 5 = 0$

Using $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = 1$, $b = 4$, & $c = 5$

$k = \frac{-4 \pm \sqrt{4^2 - 4\times1\times5}}{2\times1}$

$k = \frac{-4 \pm \sqrt{16-20}}{2}$

$k = \frac{-4 \pm \sqrt{-4}}{2}$

$\therefore k_1 = \frac{-4+2i}{2} = \frac{2(-2+i)}{2}$

$\therefore k_1 = -2+i$, $k_2 = -2-i$

$y_1 = e^{(-2+i)x}$

$y_2 = e^{(-2-i)x}$

$\Rightarrow y_1 = e^{-2x}\cdot e^{ix}$

$y_2 = e^{-2x}\cdot e^{-ix}$

$y = c_1y_1 + c_2y_2$

$y = c_1(e^{-2x}\cdot e^{ix}) + c_2(e^{-2x}\cdot e^{-

$y = e^{-2x}[C_1 e^{ix} + C_2 e^{-ix}]$

$y = e^{-2x}[C_1 \cos x + C_2 \sin x]$

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

Substitute

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{x}$

$e^{\cancel{2x}}(4A - \cancel{8A} + 5A) = 2e^{x}$

$\cancel{e^x}(A) = 2\cancel{e^x}$

$A = 2$

$\therefore y = e^{-2x}[C_1 \cos x + C_2 \sin x] + 2e^{-2x}$

at $x = 0$, $y = 1$

$1 = e^{-2(0)}[C_1 \cos(0) + C_2 \sin(0)] + 2e^{-2(0)}$

$1 = 1(C_1 + 0] + 2$

$1 = C_1 + 2$

$C_1 = -1$

To get C_2

$y' = -2e^{-2x}[-C_1 \sin x + C_2 \cos x]$
$- 4e^{-2x}$

at $x = 0$, $y = -2$

$-2 = -2e^{-2(0)}[-C_1 \sin(0) + C_2 \cos(0)]$
$- 4e^{-2(0)}$

$-2 = -2[0 + C_2] - 4$

$-2 = -2C_2 - 4$

$-2C_2 = 2$

$C_2 = -1$

$y = e^{-2x}[C_1 \cos x + C_2 \sin x)$
$+ 2e^{-2x}$

Substitute $C_1 = -1$ & $C_2 = -1$

$y = e^{-2x}[\cos x - \sin x] + 2e^{-2x}$

$y = e^{-2x}[-\cos x - \sin x + 2]$

$y = e^{-2x}[2 - \cos x - \sin x]$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Solution

$3y'' + 2y' - y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$3k^2 e^{kx} - 2ke^{kx} - e^{kx} = 0$

$e^{kx}(3k^2 - 2k - 1) = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k) + (k - 1) = 0$

$3k(k - 1) + 1(k - 1) = 0$

$(3k + 1)(k - 1) = 0$

$3k + 1 = 0$ and $k - 1 = 0$

$3k = -1$ $\quad k_2 = 1.$

$k_1 = -1/3$

$y_1 = e^{-1/3 x}$

$y_2 = e^{x}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{-1/3 x} + C_2 e^{x}$

$y_p = Ax - Bx^2$

$y' = A - 0 + 0$

$y'' = 0$

Substitute

$0 - 2A - Ax - B = 2x - 3$

$-A\cancel{x} = 2\cancel{x}$

$A = -2$ $\quad$ ~~At~~ $A = -2$,

$-2A - B = -3$

$-2(-2) - B = -3$

$4 - B = -3$

$B = 7$