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ELECT/ELECT ENG
16/ENG04/039 ENG MATHS

The parametric equation of a curve as given in equ (1) and (2)

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

In terms of t determine

(i) An expression for the radius of curvature (R), and

(ii) Expressions for the coordinates (h, k) of the centre of curvature.

(i) $R = \dfrac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2}$

$dy/dx = ?$, $d^2y/dx^2 = ?$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$

$y = \sin t - t\cos t$

$dy/dt = \cos t - [-t\sin t + \cos t]$

$= \cos t + t\sin t - \cos t$

$dy/dt = t\sin t$

$x = \cos t + t\sin t$

$dx/dt = -\sin t + [t\cos t + \sin t]$

$= -\sin t + t\cos t + \sin t$

$dx/dt = t\cos t$

$dy/dx = t\sin t \div t\cos t$

$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$= \frac{V\,{}^{du}/_{dx} - U\,{}^{dv}/_{dx}}{V^2} \times \frac{dt}{dx}$

$= \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$

$= \frac{\cos^2 + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$

$\frac{d^2y}{dx} = \frac{1}{t\cos t}$

$R = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$

$= [1 + (dy/dx)^2]^{3/2} \div d^2y/dx^2$

$= \left[\frac{1}{1} + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$

$= \left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$

$= \left[\frac{\cos^2 + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$

$= \left(\frac{1}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$

$= \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$

$= \frac{t\cos^3 t}{\cos^3 t}$

$R = t$

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