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1 $\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$

for C.F

$m^2 + 4m + 5 = 0$

Since it's an equation with complex root

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is used

$\frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$\frac{-4 \pm \sqrt{16 - 20}}{20} \gg \frac{-4 + \sqrt{-4}}{2}$

$= \frac{-4 \pm 2j}{2} \gg 2 + j$

$\therefore y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

Assumed P.I

$y = C\cos\theta + D\sin\theta$

$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) +$
$5(C\cos\theta + D\sin\theta) = 6\sin\theta$

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$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta$
$+ 5D\sin\theta = 6\sin\theta$

~~Comparing coefficients~~

$\sin\theta(-D - 4C + 5D) = 6\sin\theta$ ...... ①

$\cos\theta(-C + 4D + 5C) = 0$ ...... ②

~~$4D - 4C = 6$~~ ...... ③

~~$4C + 4D = 0$~~ ...... ④

Subtract eqn 4 from 3

$= -8C = 6$

$C = \frac{-6}{8} = -3/4$

From eqn 3

$4(-3/4) + 4D = 0$

$\frac{12}{4} = 4D$

$D = 3/4$

$y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta.$

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$y = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$ 2.

General equation = C.F + P.I

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

i $y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta)$ //

ii $\cancel{y = \frac{3}{4}(\sin\theta - \cos\theta)}$

For $\theta = 0°$ to $270°$

Consider P.I $\gg y = \frac{3}{4}(\sin\theta - \cos\theta) \gg \frac{3}{4}\sin\theta - \frac{3}{4}\cos$

$\frac{dy}{d\theta} = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

At steady state, $\frac{dy}{dx} = 0$ and θ tends towards Infinity (∞

$0 = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta \gg -\frac{3}{4}\cos\theta = \frac{3}{4}\sin\theta$

$\frac{-\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = 1$

$\theta = \tan^{-1}(-1)$

$\theta = -45$

$\theta = 315$

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2. $EI\frac{d^2y}{dx^2} = \frac{W}{2}(L-x)^2$

C.F

$m^2 = 0$

$m = \sqrt{0}$ ∴ $m = \pm 0$ twice

$y = e^0(A+Bx)$

$y = A + Bx$

P.I

$y = Cx^2 + Dx^3 + Fx^4 \dots$

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Fx^3 \dots$

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Fx^2 \dots$

Subst eqn into Original eqn

$EI(2C + 6Dx + 12Fx^2) = \frac{W}{2}(L-x)^2$

$2CEI + 6DEIx + 12FEIx^2 = \frac{W}{2}(L-x)^2$

~~$2CEI + 6DEIx + 12FEIx^2 = \frac{W}{2}(L-x)(L-x)$~~

$\qquad \text{"} \qquad = \frac{W}{2}(L^2 - 2Lx + x^2)$

Cross multiply

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$= 4CEI + 12DEIx + 24FEIx^2 = w(l^2 - 2lx + x^2)$

$4CEI + 12DEIx + 24FEIx^2 = wl^2 - 2wlx + wx^2$

Comparing Coefficients

$\cancel{x^2}(24FEI) = 2w\cancel{x^2}$ $\quad F = \dfrac{w}{24EI}$

$\cancel{x}(12DEI) = -2wl\cancel{x}$ $\quad D = \dfrac{-2wl}{12EI}$

Constant : $4CEI = wl^2$

$C = \dfrac{wl^2}{4EI}$

Putting back into the original equation P.I

$$y = \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$$

$G.E = y = C.F + P.I$

$$y = A + Bx + \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$$

$$\frac{dy}{dx} = B + \frac{2wl^2x}{4EI} - \frac{6wlx^2}{12EI} + \frac{8w}{24EI}x^3$$

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at $y = 0$ and $x = 0$

$$0 = B + \frac{\cancel{2}^{1} wl^2 x}{{}_2\cancel{4}EI} - \frac{\cancel{6} wl x^2}{\cancel{12}_2 EI} + \frac{\cancel{8}^{1} w}{\cancel{24}_3 EI} x^3$$

$$0 = B + \frac{wl^2 0}{2EI} - \frac{wl(0)^2}{2EI} + \frac{w}{3EI}(0)^3$$

$$0 = B + 0 - 0 + 0$$

$\therefore B = 0$

G.E

$$y = \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$$

$$y = \frac{wl^2}{4EI}x^2 - \frac{wl}{6EI}x^3 + \frac{w}{24EI}x^4$$

at $x = l$

$$y = \frac{wl^4}{4EI} - \frac{wl^4}{6EI} + \frac{wl^4}{24EI}$$

$$y = \frac{6wl^4 - 4wl^4 + wl^4}{24EI}$$

$$y = \frac{3wl^4}{24EI}$$

$$y = \frac{wl^4}{8EI}$$ //