

$$1. \frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

$$y'' = 4y' + 5y = 0$$

$$\text{Let } y = e^{k\theta}$$

$$y' = ke^{k\theta}$$

$$y'' = k^2e^{k\theta}$$

$$k^2e^{k\theta} + 4ke^{k\theta} + 5e^{k\theta} = 0$$

$$(k^2 + 4k + 5)e^{k\theta} = 0$$

$$(k^2 + 4k) = -5$$

$$k^2 + 4k + 2k = -5 + 2^2$$

$$k^2 + 2^2 = -5 + 2^2$$

$$(k+2)^2 = -1$$

$$k+2 = \sqrt{-1}$$

$$k = \sqrt{-1} - 2$$

$$z = \pm j - 2$$

$$= -2 \pm j$$

$$y_h = C_1 e^{-2\theta} (C_1 \cos\theta + C_2 \sin\theta)$$

$$\text{Recall } y_1 = e^{k_1\theta} = e^{(j-2)\theta} = e^{j\theta} \cdot e^{-2\theta}$$

$$y_2 = e^{k_2\theta} = e^{(j-2)\theta} = e^{j\theta} \cdot e^{-2\theta}$$

$$y_h = C_1 y_1 + C_2 y_2$$

$$= (C_1 e^{j\theta} e^{-2\theta}) + (C_2 e^{j\theta} e^{-2\theta})$$

$$y_h = e^{-2\theta} (C_1 \cos\theta + C_2 \sin\theta)$$

$$PI = y = A \sin \theta + B \cos \theta$$

$$y' = A \cos \theta - B \sin \theta$$

$$y'' = -A \sin \theta - B \cos \theta$$

$$y'' + 4y' + 5y = 6 \sin \theta$$

$$-A \sin \theta - B \cos \theta + 4(A \cos \theta - B \sin \theta) + 5(A \sin \theta + B \cos \theta) = 6 \sin \theta$$

$$-A \sin \theta - B \cos \theta + 4A \cos \theta - 4B \sin \theta + 5A \sin \theta + 5B \cos \theta = 6 \sin \theta$$

$$-A \sin \theta + 5A \sin \theta - 4B \sin \theta - B \cos \theta + 4A \cos \theta + 5B \cos \theta = 6 \sin \theta$$

$$4A \sin \theta - 4B \sin \theta + 4B \cos \theta + 4A \cos \theta = 6 \sin \theta$$

$$(4A - 4B) \sin \theta + (4B + 4A) \cos \theta = 6 \sin \theta$$

$$4A - 4B = 6 \quad \dots \textcircled{1}$$

$$4B + 4A = 0 \quad \dots \textcircled{2}$$

From equ $\textcircled{2}$

$$4B = -4A$$

$$B = -A$$

$$A = -B$$

$$4(-B) - 4B = 6$$

$$-8B = 6$$

$$B = \frac{6}{-8}$$

$$= -\frac{3}{4}$$

$$A = -(-\frac{3}{4}) = \frac{3}{4} //$$

$$PI = y = \frac{3}{4} \sin \theta - \frac{3}{4} \cos \theta$$

$$GS = e^{-2\theta} (C_1 \cos \theta + C_2 \sin \theta) + \frac{3}{4} \sin \theta - \frac{3}{4} \cos \theta$$

$$= e^{-2\theta} (A \cos \theta + B \sin \theta) + \frac{3}{4} (\sin \theta - \cos \theta)$$

ii Steady state eqn: $y' = \frac{3}{4} \sin \theta + \frac{3}{4} \cos \theta = 0$

$$\frac{\frac{3}{4} \sin \theta}{\frac{3}{4} \cos \theta} = \frac{-\frac{3}{4} \cos \theta}{\frac{3}{4} \cos \theta}$$

$$\tan \theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$= -45^\circ$$

$$2EI \frac{d^2y}{dx^2} = w(L-x)^2$$

Solution.

$$EI k^2 = 0$$

$$k^2 = 0$$

$$k = \pm \sqrt{0}$$

$$k = \pm 0$$

$$y = e^{\alpha} (A + Bx)$$

$$y = C + Dx = A + Bx$$

$$EI = y = Ex^2 + Dx^3 + Fx^4$$

$$y' = 2Cx + 3Dx^2 + 4Fx^3$$

$$y'' = 2C + 6Dx + 12Fx^2$$

$$EI(2C + 6Dx + 12Fx^2) = \frac{W}{2} (L-x)^2$$

$$2CEI + 6DxEI + 12Fx^2EI = \frac{W}{2} (L-x)^2$$

$$4CEI + 12DxEI + 24Fx^2EI = W(L^2 - 2Lx + x^2)$$

$$24FEI = W$$

$$F = \frac{W}{24EI}$$

$$12DEI = 2WL$$

$$D = \frac{2WL}{12EI}$$

$$= \frac{WL}{6EI}$$

$$6EI$$

$$4CEI = WL^2$$

$$C = \frac{WL^2}{4EI}$$

$$4EI$$

$$y = \delta = \left[\frac{WL}{4EI} \right] x^2 - \left[\frac{WL}{6EI} \right] x^3 + \left[\frac{W}{24EI} \right] x^4$$

$$= \frac{WL^2 x^2}{4EI} - \frac{WLx^3}{6EI} + \frac{Wx^4}{24EI}$$

$$y = \frac{6WL^2 x^2 - 4WLx^3 + Wx^4}{24EI}$$

$$\delta = A + Bx + \frac{W}{24EI} [6L^2 x^2 - 4Lx^3 + x^4]$$

at $y=0$, $x=L$ and $\frac{dy}{dx} = 0$

$$0 = A + B(L) + \frac{W}{24EI} [6L^2(L)^2 - 4(L)(L)^3 + (L)^4]$$

$$0 = A + B(L) + \frac{W}{24EI} (L)$$

$$0 = A + 0 + 0$$

$$\therefore A = 0$$

$$\frac{dy}{dx} = B + \frac{W}{24EI} (12L^2 x - 12Lx^2 + 4x^3)$$

$$0 = B + \frac{W}{24EI} (12L^2(L) - 12L(L) + 4(L))$$

$$0 = B + W (w)$$

$$24EI$$

$$B = 0$$

$$y = \frac{w}{24EI} (6L^2x^2 - 4Lx^3 + x^4)$$

$$24EI$$

$$y = \frac{w}{24EI} \cdot x^2 (6L^2 - 4Lx + x^2)$$

$$24EI$$

When $x = L$

$$y = \frac{w}{24EI} \cdot L^2 (6L^2 - 4L^2 + L^2)$$

$$24EI$$

$$y = \frac{wL^2}{24EI} \cdot (3L^2)$$

$$24EI$$

$$y = \frac{3wL^4}{24EI}$$

$$24EI$$

$$y = \frac{wL^4}{8EI}$$

$$8EI //$$

	A	B	C	D	E	F	G	H
1	teta	y						
2		0	-0.75					
3		9	0.99243656					
4		18	-1.058478					
5		27	0.93638605					
6		36	-0.6478614					
7		45	0.24418615					
8		54	0.20289059					
9		63	-0.6139057					
10		72	0.91580546					
11		81	-1.0549305					
12		90	1.00655271					
13		99	-0.7792708					
14		108	0.41348168					
15		117	0.02579944					
16		126	-0.460495					
17		135	0.81334239					
18		144	-1.0216267					
19		153	1.0483277					
20		162	-0.8886994					
21		171	0.5711142					
22		180	-0.1520194					
23		189	-0.2940952					
24		198	0.6879375					
25		207	-0.9595062					
26		216	1.06053268					
27		225	-0.9730607					
28		234	0.71263739					
29		243	-0.3255503					
30		252	-0.1193999					
31		261	0.54312807					
32		270	-0.8703209					

