NYE UCHE CHUKS
15/ENG04/037
ENG 331

ASS **Pt 1**

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Soln

By Taking the left hand side an equating it to zero

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

The auxilliary equation becomes

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$m(m-2) + 1(m-2) = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1 \text{ or } m = 2$$

$$y = Ae^{-x} + Be^{2x}$$

The assume PI

$$f(x) = 8$$

$$y = c \quad \text{---} \quad ①$$

$$\frac{dy}{dx} = 0 \quad \text{---} \quad ②$$

$$\frac{d^2y}{dx} = 0 \quad \text{---} \quad ③$$

By substituting equation 1, 2 and 3 into the original equation

$$0 - 0 - 2(c) = 8$$

$$-2c = 8$$

$$c = \frac{8}{-2}$$

$$c = \underline{-4}$$

The General Solution = Complementary function + Assumed P.I

$$y = Ae^{-x} + Be^{2x} - 4$$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

To find C.F

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$$y = Ae^{2x} + Be^{-2x}$$

Assumed P.I

$f(x) = 10e^{3x}$

$y = Ce^{3x}$ --- (1)

$\frac{dy}{dx} = 3Ce^{3x}$ --- (2)

$\frac{d^2y}{dx^2} = 9Ce^{3x}$ --- (3)

By Substituting equation 1, 2 and 3 into the original equation

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5C\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$C = \frac{10}{5}$

$C = 2$

Assumed Pi

$y = 2e^{3x}$

The General Solution = C.F + P.I

$$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$$

② $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

To find C.F

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

The auxilliary equation becomes

$m^2 + 2m + 1 = 0$

$m = -1$ (twice)

$y = e^{-x}[A + Bx]$

To find assumed P I

$f(x) = e^{-2x}$

$y = Ce^{-2x}$ --- ①

$\frac{dy}{dx} = -2Ce^{-2x}$ --- ②

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$ --- ③

By Substituting equation 1, 2, and 3 into the original equation

$4Ce^{-2x} + 2[-2Ce^{-2x}] + Ce^{-2x} = e^{-2x}$

$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$

$C\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$C = 1$

$y = e^{-2x}$

The general Solution = C.F + P I

$y = e^{-x}[A + Bx] + e^{-2x}$

(5) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Soln

To find C.F

$\frac{d^2y}{dx^2} + 25y = 0$

The auxilliary equation becomes

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-1} \times \sqrt{25}$

$m = \pm j5$

Recall

$m = \alpha \pm j\beta$

$y = A\cos 5x + B\sin 5x$

Assumed P.I

$fx = 5x^2 + x$, $y = Cx^2 + Dx + E$ ---- (1)

$\frac{dy}{dx} = 2Cx + D$ --- (2)

$\frac{d^2y}{dx^2} = 2C$ --- (3)

By substituting equation 1, and 3 into the original equation

$2C - 2[Cx^2 + Dx + E] = 5x^2 + x$

$2C - 2Cx^2 - 2Dx - 2E = 5x^2 + x$

By Comparing Coefficients

x^2: $2C = 5$

$C = \frac{5}{2}$

x: $-2D = 1$

$D = \frac{-1}{2}$

K: $2C - 2E = 0$

$2\left(\frac{5}{2}\right) - 2E = 0$

$5 = 2E$

$E = \frac{5}{2}$

$y = \frac{5x^2}{2} - \frac{x}{2} + \frac{5}{2}$

The General Solution $= A\cos 5x + B\sin x + \frac{5x^2}{2} - \frac{x}{2} + \frac{5}{2}$

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ENG 281

5) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$ RSS P12

CF $m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m_1 = 4, \; m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

PI Assume $y = Cxe^{4x}$

$\frac{dy}{dx} = [x \cdot 4e^{4x} + e^{4x}]$

$\frac{d^2y}{dx^2} = 4c[x4e^{4x} + e^{4x}] + 4ce^{4x}$

$= 16cxe^{4x} + 4ce^{4x} + 4ce^{4x}$

$= 16cxe^{4x} + 8ce^{4x} - 6[4cxe^{4x} + ce^{4x}] + 8cxe^{4x} = 8e^{4x}$

$16cx^{4x} + 8ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8cxe^{4x} = 8e^{4x}$

$16cx + 8c - 24cx - 6c + 8cx = 8$

$= -8cx + 2c + 8cx = 8$

$2c = 8$

$c = \frac{8}{2}$

$c = 4$

$y = 4xe^{4x}$

$GS = CF + PI$

$= \underline{Ae^{4x} + Be^{2x} + 4xe^{4x}}$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-x}$

Given that at $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

CF $m^2 + 4m + 5 = 0$

This is a complex situation

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm j2}{2}$

$= -2 \pm j$

$m_1 = -2 + j$, $m_2 = -2 - j$

$y = e^{-2x}(A\cos x + B\sin x)$

PI Assume $y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$

$\frac{d^2y}{dx^2} = 4ce^{-2x}$

$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$

$4c - 8c + 5c = 2$

$c = 2$

$y = 2e^{-2x}$

G.S => $e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$e^{-2x}[-A\sin x + B\cos x] + [A\cos x + B\sin x] - 2e^{-2x}$
$+ 2e^{2x}$

$A\sin x + B\cos x] - 2e^{-2x}[A\cos x + B\sin x] - 4e^{2x}$

at $x = 0$, $\frac{dy}{dx} = -2$

$-2 = e^{-2(0)}[+A\sin 0 + B\cos 0] - 2e^{-2(0)}[A\cos 0 + B\sin 0] \ldots$

$-2 = B - 2A - 4$

$B = -2 - 2 + 4$

$B = 0$

$\therefore$ PS $= e^{-2x}(-\cos x) + 2e^{-2x}$

$= e^{-2x}\cos x + 2e^{-2x}$

$= e^{-2x}(2 - \cos x)$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

CF $3m^2 - 2m - 1 = 0$

$(3m + 1)(m - 1) = 0$

$m_1 = -\frac{1}{3}$ $m_2 = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$

P.I Assume $y = (cx + D)$

$\frac{dy}{dx} = c$

$\frac{d^2y}{dx^2} = 0$

$3(0) + 2(0) - [(cx + D)]^2 = 2x - 3$

$0 - 2c - (cx - D) = 2x - 3$

x: $-c = 2$

$c = -2$

Constants $-2(-D) = -3$

$4 - D = -3$

$D = 7$

$y = -2x + 7$

G.S => $Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

(3) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

The auxillary equation becomes

$m^2 - 6m + 8 = 0$

$m = 4$ or $m = 2$

$y = \underline{Ae^{4x} + Be^{2x}}$